● *ABSTRACT*

The concepts of game theory are analyzed to determine their selective usefulness for studying certain types of problems and obtaining certain types of conclusions. It is found that game theory lends itself to perspectives and applications which do not question the assumptions underlying existing structures or institutions in society. The selective usefulness of game theory concepts is reflected in the actual uses and applications of the theory; several examples are given. Furthermore, this selective usefulness is quite compatible with what is known about the origins of game theory. The study of game theory concepts provides a useful example of a perspective on mathematics from which mathematical frameworks, as well as associated concepts, are seen as being value-laden.

The Selective Usefulness of Game Theory

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There are now several studies showing how the content of scientific knowledge may be shaped by political, economic, social, intellectual, or other 'non-scientific' influences[1]. There are also studies which show that scientific knowledge (often that produced under the influence of such 'non-scientific' factors) can selectively lend itself to certain practical or ideological uses.[2] For example, the concepts underlying the Darwinian theory of evolution (survival of the fittest, the struggle for existence) may be traced to prior concepts used to explain capitalist society; the concepts of evolution in turn lent themselves to a justification of social policies which reinforced this type of social organization (Social Darwinism).[3]

This latter type of influence — the influence of scientific knowledge on social and political development — may be interpreted in at least two ways. First, it may be argued that the observations and hypotheses underlying the scientific knowledge are largely neutral and innocuous in themselves, and that is the interpretations which are given to the empirical evidence and to the

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mathematical or logical formulations involved which make the scientific knowledge selectively useful for specific purposes in the wider society. From this perspective then, the underlying base of empirical observations and logical or mathematical formulations might be used for many purposes, depending on who tries to use it and on the social context, while it is the interpretation of these observations and formulations — explanatory metaphors, suggestive terminology, and working concepts — which determine what the scientific knowledge is most easily used for in practice. Many of the cases which have been studied in this context seem to be interpretable in this way. While it is easy to see how Darwin's theory owed its practical interpretation to the Malthusian theory of society, it is less easy to see how Darwin's observations of plants and animals actually forced a certain interpretation of evolution (although his framework of ideas would seem to lend itself to Social Darwinist uses). While it is easy to see how Francis Galton's statistical ideas were influenced by his political commitment to eugenics,[4] it is less easy to see how regression analysis and other statistical tools which he developed are selectively useful for particular social or political applications. Again, while a plausible case can be made for the influence of anti-rationalism in Weimar Germany on the origins and formulation of quantum mechanics,[5] it is much more difficult to demonstrate how scientists' observations of quantum phenomena and the mathematical framework of quantum mechanics lend themselves to interpretations from particular intellectual positions — especially when there are now a range of interpretations of quantum mechanics which are being advocated.

Nevertheless, it is possible to interpret the influence of scientific knowledge on social and political development in a second way, by tracing it to the very essence of the scientific knowledge. Adopting the framework of paradigms[6] and the theory-ladenness of observations,[7] it may be argued that observations — because their existence and formulation depend on theoretical frameworks utilizing concepts from the wider society — selectively lend themselves to particular uses in society. Even more so than observations, mathematical or logical formulations used in science may be closely tied to special ways of interpreting the world, and so selectively lend themselves to particular uses in society.

In this paper one particular framework of mathematics and ideas— game theory — will be described and interpreted from the latter perspective. Namely, it will be argued that while the mathematics and concepts of game theory *permit* a wide range of applications, at the same time they are selectively useful in solving problems of a certain type, in drawing attention to certain features of a problem, and in reaching certain types of conclusions. As it happens, game theory lends itself to applications which do not question the assumptions underlying existing structures or institutions (economics, politics, interpersonal relations), and is difficult to use in problems involving changes in structures or institutions. Furthermore, this selective usefulness is what might be expected considering what is known about the origins of game theory.

These conclusions perhaps are not surprising; the value of such a study of game theory is that the links between the selective usefulness of the theory and basic features of the mathematics are fairly clear. Studying such links might be more difficult in other areas — such as the kinetic theory of gases or topology. The advantages of studying game theory in such an endeavour are several:

(1) Game theory is very recent, so that its diversification, transformation, and institutionalization in scientific theory and practice have not proceeded as far as in the case of other branches of mathematics and science.

(2) Game theory is almost solely the creation of one person — John von Neumann — and so stands relatively coherent and free from doctrinal conflict.

(3) Game theory is concerned with an area — conflict situations — in which there is no accepted framework of relevant evidence and interpretation; therefore the impact of a new theory on the direction of scholarly attention is clearer.

Perhaps it will be useful to spell out in advance that the analysis made here is not meant to follow the conventional 'use/abuse' framework. For the sake of illustration, consider two positions concerning the moral status or 'bias' of tools. One is that tools are neutral, and that any unfortunate consequences due to using them are solely the responsibility of the user. The second position holds that it is reasonable to blame, to some extent, the tool itself (or more precisely, those people who developed and promoted it) for such unfortunate consequences. To take an extreme example: imagine a stereo amplifier designed in such a way that if one touches a certain button, the amplifier violently explodes. The consequent series of 'accidents' might be blamed on the carelessness of users; but most people would find it more reasonable to blame the designer and promoter. Real life examples are never this simple. But my analysis of game theory is based on the idea that mathematics, seen as a humanly constructed tool, is some distance away from pure neutrality and along the road towards the position of the amplifier (or other tools lending themselves instead to praiseworthy occurrences): some of the praise or blame for the consequences of the application of mathematical models should be laid at the door of originators, developers, and promoters.[8]

One result of this perspective is that it becomes natural to apply moral judgements to mathematics — which may be both unfamiliar and uncomfortable for mathematicians. Such judgements are not meant only for game theory; the analysis here is meant to be illustrative of an analysis of selective usefulness that might be applied to any part of mathematics.

In the next Section a simple introduction to the concepts of game theory will be given. In Section 2 a critique of some of the basic concepts of the theory will be presented, highlighting the values intrinsic to these concepts. In Section 3 the applications and uses of game theory are reviewed briefly, and several particular applications analyzed in more detail. These applications and their limitations nicely reflect the values in the game theory concepts. In Section 4 a few comments on the origin of game theory are made; this origin is quite compatible with the values and applications of the theory. In Section 5 an attempt is made to treat the question of whether the values in game theory concepts are also reflected in the mathematical formalism. The answer depends on one's perspective; in the perspective favoured here it is useful to associate values with the mathematical formalism. In the final section a few comments about interpreting game theory are made.

1. SOME CONCEPTS OF GAME THEORY

The basic concepts of game theory will be approached here in an intuitive manner, rather than in a formal or mathematical sense. Those who prefer a more rigorous treatment can resort to a range of excellent accounts.[9]

Let's start by taking a number of gaming or bargaining or conflict situations, such as (1) chess, (2) war, (3) a couple deciding where to go Saturday night, (4) selling or buying goods, (5) a duel, and (6) peace negotiations. These examples have been selected for a purpose. They have in common a number of features: (1) conflicting parties ('players'), such as chess opponents; (2) choices, such as what military tactic to adopt; (3) information, such as where people might go Saturday night; (4) desired results, such as a high price for goods sold; (5) results of choices, such as death or victory in a duel (or ignominy, from running away); and (6) the outcome being dependent on the actions of all participants, so that nobody can force an outcome, as is obviously the case in peace negotiations.

These features provide the essence of a theoretical model for conflict situations: two or more players have a range of action or freedom equivalent to a set of choices, and have certain information. Each player has a set of preferences for the different possible outcomes, and the results of the interaction depend on all the players' decisions.

Now let's take an example of possible actions that might have been taken by Hanoi and Washington during the Vietnam war. Assume that Washington has three choices — escalate, negotiate, or pull out — and that Hanoi can either escalate or negotiate. Assume also a set of outcomes corresponding to each set of choices: for example, if both sides escalate there will be military stalemate but much more killing. The features of the conflict situation might be represented in a matrix like the following:-

		Washington		
		escalate	negotiate	pull out
Hanoi	escalate	military stalemate; more killing	military advantage to Hanoi	military victory to Hanoi
	negotiate	military advantage to Washington	military and political stalemate; less killing	political advantage to Hanoi

This matrix includes the players (Washington and Hanoi), choices (escalate, etc.), outcomes or results (military statemate, etc.), and the dependence of the outcome on the actions of each player. It does not show the information possessed by each side (Washington might believe that escalation would bring military victory), nor the results desired by either player. Before much progress can be made on the problem mathematically, it is necessary to assume that all players have complete information about choices and outcomes, and to assume the existence of a numerical preference scheme for comparing the values of the outcomes. Using some (rather arbitrary) figures to compare the value of the different outcomes to each player, we might say that military stalemate with more killing was worth -1 to Hanoi and -2 to Washington, and so forth, leading to the following matrix:-

		Washington		
		escalate	negotiate	pull out
Hanoi	escalate	(-1, -2)	(1, -3)	(3, -5)
	negotiate	(-3, 1)	(0, 0)	(3, -3)

The first number in each pair is the payoff to Hanoi, the second the payoff to Washington.

In the cases where the mathematical techniques of game theory can be applied most conclusively and elegantly, the preferences of the players must be exactly opposed, as is the case in the lower right hand box above (3, -3). This idealization is sometimes a good first approximation (as in duels between airplanes or in parlour games) and forms the basis for understanding the more complex situations. Rewriting the above matrix with only a single payoff (as near as possible representing the payoffs above), and using abstract symbols for the players and choices, we obtain

		A		
		A1	A2	A3
B	B1	0	2	4
	B2	-1	0	3

If player A chooses A2 and player B chooses B1, then the payoff to B is 2 and the payoff to A is -2. This is called a zero-sum game, because any gain to A is a loss to B, and vice versa.

If we assume that each player has a goal, then we may attempt to prescribe actions which will realise it. Game theory prescribes courses of action for the attainment of outcomes which have certain formal 'optimum' properties. It does not say what a person 'should' do in an absolute sense; it is not descriptive, but rather 'conditionally normative.' Nevertheless, the mathematical techniques used in game theory are geared to the achievement of a single goal: maximization of the 'security level', where the security level is the least amount a player can receive from a strategy choice. The solution (set of expected outcomes) to the game when this strategy is adopted by each player is the 'equilibrium solution', so-called because neither player can gain by changing her/his strategy unless the other player also changes her/his strategy.

It is at this stage that the mathematical aspects of game theory begin. Solutions for two-person, zero-sum games with finite numbers of choices are straightforward. The mathematicians carry on to n-person games, infinite games, non-zero-sum games, existence and uniqueness of solutions, the extensive form and the characteristic function form, as well as many other refinements, variations, and puzzles. Much of this work is fascinating; here I have not even mentioned the simple and crucial idea of a 'mixed strategy'. This is because my aim has been mainly to introduce the concepts underlying game theory, and to illustrate the links these concepts provide between real-life situations and the abstract formulation of a problem as it might be approached by a mathematician.

2. VALUES BUILT INTO THE CONCEPTS OF GAME THEORY

It is my contention that the concepts which provide the basis for game theory are not 'neutral' in any useful sense. That is, these concepts and the mathematical theory based on them lend themselves to the study of certain types of problems, lend themselves to an emphasis on certain aspects of any problem studied, and lend themselves to certain types of solutions. The values embedded in game theory concepts lead to the selective usefulness of the theory: it can be used easily for some purposes, and only with the greatest difficulty for others.

The critique of concepts here should not be construed as a criticism of simplification per se. In any process of mathematical modelling, simplification is a necessity. The important point is that there are innumerable ways in which a given situation may be simplified, depending on which features are to be emphasized and which are to be ignored. Values enter through the decision to make a particular set of simplifications and to build a mathematical formalism based upon these simplifications. So in simplifying a situation, one inevitably introduces a bias; the question is, *which* bias? The analysis of game theory concepts here is meant to delineate the special features of the world selected for attention by the simplification that constitutes the basis for the game theory model of reality. The values that are associated with the particular simplifications in game theory are suggested by the values built into game theory concepts themselves, and also by the preferred applications of the theory and the circumstances surrounding the origin and promotion of the theory. It is to the concepts then that attention is turned in this section.

The concepts of game theory scrutinized here are 'player', 'choice', and 'payoff'. The general conclusion will be that these concepts can be expected to lend themselves to the study of situations from a viewpoint of individualism and competitiveness within a 'refied representation of the status quo' (that is, a model of reality which incorporates the assumptions of the model-builder, and ensures that those built-in assumptions are fixed for all uses of the model).

'Player'

Firstly, the players in a game are generally seen as opponents. Each player is concerned only to maximize her/his payoff. (If a player is altruistic and values a large payoff to another player, the payoffs are adjusted so that the altruistic player is only concerned about her/his payoff.) The idea of the player makes it easier to apply game theory to competitive situations, and to situations based on individualistic ethics. This bias is enhanced by the mathematical tractability of zero-sum games, where the interests of the players are diametrically opposed. So although in principle game theory can treat cooperative situations (non-zero-sum games), it lends itself most easily to treatment of competitive situations.[10]

Secondly, the players in a game are fixed. If situations with changing coalitions are to be treated, then all possible separable coalition elements must be included as players. But this means that the difficult n-person game theory must be used. The *tendency* is to keep the number of players small, and if possible limit it to two.[11] If a 'player' is an organization or country, this causes diverse and conflicting interests within the organization or country to be submerged. Obtaining solutions that involve a realignment of interests is thereby hindered by the static 'player' concept.

'Choice'[12]

As in the case of 'player', in a game the choices between which a player may choose are generally conceived as fixed. This means that new choices (for example choices arising as a result of bargaining or change of values) are not readily included in the application of game theory. Because a large number of choices makes practical analysis of a game more difficult, the *tendency* is to reduce the number of choices in an analysis to the minimum number possible. Once again this means that new and unusual options tend to be downgraded.

The use of fixed choices tends to reify the status quo of acceptable actions. Humanly constructed situations — such as current military intervention, legal precedent, or standard business practices — are given a significance that makes them seem to be part of the intrinsic essence of things. The idea of choices as fixed options makes it easy to ignore the human construction of past circumstances, and easy to forget that future options will be similarly constructed.

'Payoff'

The payoffs which accrue to each player as a result of their choices are made in some common measure (such as money) which, if the mathematical analysis of game theory is to be used at all, must be translated into numbers. Game theory therefore is most easily used in situations in which people's values are quantified. Apparently disparate values — such as those associated with human lives, material possessions, prestige, and joy — must be reduced to a common measuring stick. Because these factors have to be reduced to numbers, the tendency is to choose quantifiable values — those that lend themselves to quantitative evaluations — and therefore to downgrade spiritual, ethical, aesthetic, and cultural factors in an analysis.

Payoffs are normally conceived of as fixed. This eliminates the possibility of representing changes of values, or the interaction of values with particular configurations of players or choices. As in the case of choices, the fixing of payoffs tends to reify the values of the status quo, and hide the possibility of the emergence or creation of new criteria for action in a given situation.

There are a number of other concepts and assumptions in game theory — such as the assumptions about 'rationality' and about 'information' — that could be analyzed in a similar manner — and with a similar result. But I think the basic point is clear: by the nature of its founding concepts, game theory lends itself to some practical or ideological purposes and not to others. In particular, game theory lends itself to analysis of situations which are based on individualistic and competitive ethics, and which reflect the values of the status quo. At the same time, the use of game theory to analyze any given sitation tends to force it into this sort of mould.

This is not to say that game theory *cannot* treat situations involving conflicts of interest within a 'player', involving choices which arise out of changing values and cooperative behaviour, or involving transformation of a conflict situation by human actions. It is possible that suitable modifications or extensions of game theory could be used to study such situations. But to do this would be both difficult and roundabout. Game theory lends itself to tackling certain types of problems from particular perspectives; for very different sorts of problems and perspectives, it would be more straightforward to start an analysis from a different basis, even if game theory could be contorted to apply to them.

3. APPLICATIONS OF GAME THEORY

The situations to which game theory has actually been applied reflect its selective usefulness for problems and solutions of an individualistic and competitive nature, building in the values of the status quo. The two principal areas of application have been war and economics. For the military it has been applied to tactical decision-making (in particular via the theory of differential games) and in studying global nuclear strategies such as deterrence. In economics, game theory has been used in studying competition for markets, advertising, planning under uncertainty, and so forth. These primary areas of application — war and economics — are where one would expect game theory to be applied, given the values reflected in its concepts.

Game theory has also been applied to many other fields, such as law, ethics, sociology, biology, and of course parlour games. In all these applications, a close study of the formulation of the problem in the game theory perspective shows a strong inclination to work from existing values, consider only currently contending parties and options, and in other ways to exclude significant redefinitions of the problems at hand. Presently I will give examples of this inclination, but first it is worth mentioning the principal *uses* of game theory.

Although game theory has been applied to many situations, it has not been particularly fruitful — at least in terms of its original promise. I see at least three ways in which game theory has proved 'useful'. First, it has to led to practical advice on tactical decision-making in certain well-defined situations, especially in military areas involving missile tracking and similar tasks[13] (where the theory of differential games has led to results equivalent to control theory).[14] Second, it has provided an occupation and amusement for thousands of government bureaucrats, mathematicians, psychologists, and others who have found plenty of funds to study game theory, develop its mathematical ramifications, and play around with bargaining and simulation games.[15] Third, it has provided a perspective for looking at military and political choices that builds in many values of the status quo, that can be adapted to give nearly any results desired,[16] and which has the appearance of mathematical sophistication. Game theory formulations therefore serve admirably as ex post facto justifications for any decisions or policies that may be adopted by military or political élites.[17]

The values built into game theory concepts thus seem to be closely reflected in its areas of primary application (war and economics) and in what it has actually been used for (tactical decision-making, employment of people studying game theory, legitimizing military and political decisions).

Until now the impression may have been given that game theory is primarily used to represent (for academics) the way in which decisions are made. But in many cases game theory is used as a tool by certain people who are actually in these sitations. In these applications, the ostensible reason for applying game theory is to obtain insights concerning what policies should be adopted by particular actors.[18] One important actual result of such applications, though, is an implicit justification and reinforcement of the assumptions which are built into the game theory formulation itself. That is, by specifying a limited range of potential action, game theory formulations encourage a perception that these actions are the only feasible or rational ones.

The following case studies illustrate the type of narrow perspective that inevitably seems to arise when game theory is applied to a problem situation. The areas I briefly comment on are international relations, ethics, and crime. Applications in other areas are similarly limited.[19]

3.1 International Relations

Due to the selective usefulness of game theory concepts, when putting international relations into a game theory formulation it is very easy to build in nationalistic bias. Many of the discussions of international relations seem to take this form: policy or strategic decisions are assumed or made, for whatever reason (such as political factors internal to a country, vested interests of a branch of the military, and so on); then a game theory formulation of the situation is drawn up which, conveniently, gives the desired result. The game theory formulation then serves to legitimize the decision taken (or at the very least to legitimize the assumptions underlying the decision) on 'scientific' grounds.

For example, take the following representation of international affairs:[20]

		Soviet Union		
		No attack	Nuclear attack on US	Ground attack on Western Europe
United States	No response	0	-500	-100
	Massive response	0	-400	-400
	Ground force response	0	0	-150

The assumptions built into the formulation are fairly clear. Firstly, it assumes that the Soviet Union and the United States are unified entities, whereas actually each one contains diverse interests and motivations (arms industries, peace groups, personalities of decision-making élites, commercial interests, bureaucracies, and the like). Secondly, it assumes a limited range of alternatives, excluding for example unilateral disarmament and organizing the people for civilian non-cooperation. (Note the asymmetry of choices based on the assumption that the Soviet Union attacks and the US responds, nicely hiding the possibility[21] that the actual state of affairs is closer to the opposite. Snyder of course is from the US!) Thirdly, whatever payoffs are included are quite arbitrary — certainly they are not empirically based — and may simply serve to justify the policy preferred by the analyst before the game theory formulation was prepared.[22]

The same assumptions are apparent in my 'Washington-Hanoi game' used to introduce the concepts of game theory. I purposely chose this example because the assumptions involved — such as that the primary decision-makers were 'Washington' and 'Hanoi', that the struggle was fundamentally military, and especially that the lives and aspirations of the Vietnamese people are pretty much irrelevant — are particularly blatant and even obnoxious to many, and therefore more readily questioned. Yet this example is typical of the use of game theory ideas for 'studying' international relations.[23]

3.2 Ethics

In the most well-known attempt to apply game theory to ethics, Braithwaite considers the problem of equitable distribution.[24] For the two-person cooperative game Braithwaite proposes a method of solution which seems to weigh each player's ethical claims in a reasonable manner. What is more dubious is whether such an intricate adjudication of competing claims is ever necessary. A game theory formulation of an ethical problem, with a Braithwaite-type solution, may actually obscure the possibility of more basically corrective approaches to ethical problems. In game theory terms, it might be better to inquire first whether it is possible to change the players, the choices, the circumstances which determine the payoffs, and so forth, before putting the problem in a game theory format.

The example used by Braithwaite to launch his development reflects the unnecessary fixities of game theory formulations. I summarize it here, with my comments in parentheses. Suppose: Luke and Mathew are bachelors living in individual rooms in a house converted to flats by an architect who ignored acoustics [presumably it is out of the question to institute soundproofing]; each can hear anything louder than a conversation in the other's flat [why not wear earplugs when there's something noisy going on?]; it is legally impossible for either to stop the other from making as much noise as desired [why not change the law?; and why think of resorting to it when cooperation will avoid trouble?]; it is economically or sociologically impossible to move elsewhere [let's change the economic and social structure]; each has 21.00 to 22.00 hours for recreation, and it is impossible to change this [unrealistic!]; Luke plays and prefers classical piano, Mathew jazz trumpet; tonight's playing, or lack of it, does not affect the next day's preferences for playing.

The basis for the conflict is that each prefers to play his instrument and type of music, but if both play then total welfare is reduced. The artificiality of the example, as well as the limitations of any game theory solution within such restrictions, should be obvious. This is not to say that Braithwaite's adjudication method has no application, but that applying a game theory framework to ethical situations is more likely to obscure satisfactory solutions than reveal them.

3.3 Crime

As a last example of how applying a game theory formulation to a problem situation reifies the values of the fomulator, there is the application of game theory (and information theory) to the study of crime.[25] This application takes the expected path: the players are criminals and police; the choices are (for criminals) different places to rob, frequency of operation, and size of target, and (for police) different patrolling schedules; the payoffs are the size of the haul and the capture or escape of the criminal. The limitations of such an approach are numerous: it ignores collusion between police and criminals; it ignores the possibility of value change (for example, decrease in materialism) or structural change (equitable distribution of wealth) in society leading to decreased criminal activity; it defines criminal activity as what is discouraged now by police (robbery, murder) and ignores structural violence in society (poverty, war, racism), not to mention crime by other classes or occupations (for example, white collar crime);[26] it assumes the continuance of present laws (for example against abortion, alcoholism, or loitering) which may be unnecessary or unjust;[27] and so on.

Game theory as applied to crime, at least in this instance, does not lead to a real elucidation of the problems in a deep way. It may serve as a tool for police or criminals, but more importantly it serves as a mathematical, esoteric way of perpetuating and justifying existing concepts about crime.

4. THE ORIGIN AND PROMOTION OF GAME THEORY

Although in analyzing the selective usefulness of a theory it is not necessary to know how the theory came to exist, it is nice to be able to find consistency in the origin and use of the theory: to find that the values underlying the purposes for which the theory was created are similar to the values underlying the applications and uses it actually finds. The limited evidence available on the origins of game theory is fully compatible with such a consistency.

Aside from a few preliminary (but basic) mathematical studies in the 1920s,[28] game theory was first presented to the public and the academic world in a full-blown exhaustive treatment, the monumental treatise by John von Neumann and Oskar Morgenstern, *Theory of Games and Economic Behavior,* published in 1943.[29] As far as can be told from statements in this book, game theory was seen by the authors as a mathematical approach to problems which were currently outstanding in economics. Prior mathematical methods had only been able to treat the ideal market of many small producers or sellers in free competition (or the opposite simple case of complete monopoly). For these cases the optimum behaviour for the entrepreneur (or monopolist) was clear: produce at the value or sell at the price which produced the greatest profit.

But for cases intermediate between the ideal market and monopoly — namely bilateral monopoly, oligopoly, and so on — there was no suitable mathematical framework, since each participant in such an economic process affected the market by its actions; a suitable strategy for maximizing utility (profit) would have to take into account the responses of others to one's own actions. Game theory was seen by its originators as a mathematical approach to these hitherto untreated problems: 'a discussion of some fundamental questions of economic theory which require a treatment different from that which they have found thus far in the literature'.[30]

From this it is apparent that game theory was not developed as a challenge to existing economic practices or structures. Rather, it was developed as a mathematical characterization of aspects of the current economic system, and as a means of helping these features to work more efficiently in their own terms. Therefore it is not surprising that the concepts of game theory should reflect the values of the current (early twentieth-century) economic system.[31]

Of course matters are not as simple as this. For example, some of the mathematical structures of game theory may have been favoured because of their mathematical elegance. It may be that the economic 'motivation' for game theory arose as a result of the application of mathematical ideas originated for other reasons. Without looking inside the head of von Neumann (who created the fundamental mathematical ideas) it is impossible to say for sure what motivated the original mathematical ideas. All that can be said is that available evidence about motivating factors is quite compatible with the values built into game theory concepts.

Later interpretations of the reasons for developing game theory are quite compatible with this interpretation. In particular Morgenstern's strong bent towards militaristic thinking in simplistic terms[32] seems consonant with biases in the structure of game theory. But later evaluations of origins must be approached carefully, because they may reflect an unconscious justification of the direction of further applications of the theory.

It is also satisfying to find that the values of the promoters of a theory are consistent with the selective usefulnesss of the theory. Such a consistency is evident from the strong military backing for the study of game theory ever since the von Neumann-Morgenstern treatise.[33] The motivation for the development of the theory of differential games is unambiguously stated by its originator Rufus Isaacs to be military problems;[34] in addition Isaacs was employed over the years when the theory was developed (though not to its fruition) by the Rand Corporation, then devoted largely to military problems.

5. THE MATHEMATICAL FRAMEWORK OF GAME THEORY: NEUTRAL OR BIASED?

I have outlined some of the values inherent in the concepts of game theory. Are these values also inherent in the actual mathematical framework — the mathematical constructs, theorems, methods of solution, and so on — or is the mathematical side a relatively 'pure' basis which is overlaid with the value-laden concepts such as 'player', 'choice', and 'payoff'? The answer to this question depends very much on the approach one takes to the mathematics. I will describe two approaches here, and argue in favour of the one which leads to the interpretation that the mathematical framework is itself value-laden.

One approach is to look at the mathematics first, and to see what uses are likely to be made of it. If these likely uses are strongly oriented in certain directions, then it makes sense to call the mathematics biased in itself. Take for example[35] the following game formulation:

		A: A1	A: A2
B	B1	3	-1
B	B2	-2	2

The 'maxmin' solution prescribed by game theory is a strategy for B that guarantees the same return no matter what A does. Let x be the average payoff to player B, f_1 be the probability of making choice B1 and f_2 be the probability of making choice B2. Then

$$x = 3f_1 - 2f_2 = -f_1 + 2f_2 \qquad (*)$$

and $f_1 + f_2 = 1$, since some choice must be made. (That is, player B should choose B1 and B 2 each with probability ½).

If we look solely at equations such as (*) (more generally at a system of simultaneous linear equations), there does not seem to be much bias. Equations such as (*) are used to solve many varieties of problems, and only a few are selectively useful in the same way as problems in a game theory formulation. From this perspective, then, it would be sensible to deny that the values in game theory concepts carry over to any extent to the mathematics of game theory. (This is not to say that systems of simultaneous equations are not selectively useful for certain types of problems — they are — that this selective usefulness is not solely a result of the use of these equations for modelling game theory problems.)

This approach to the mathematical framework of game theory may be criticized on the basis that it assumes that mathematics arose independently of the need to solve particular problems. If it is found that mathematical ideas, techniques, theoretical frameworks, and directions of study are linked to prevalent cultural values, economic and political exigencies, or ideologies,[36] then to look first at the mathematics is to forget its origins in value-laden situations.

An analogy may be drawn between mathematical concepts and electronic circuitry. A resistor or capacitor may be seen as 'neutral' in the same sense that the 'x' in equation (*) can be seen as 'neutral'. A resistor can be used in many different situations, from missiles to heart-lung machines. But to call the *circuitry* 'neutral' in any of these particular circumstances may be misleading, because in each case the circuitry is organized in a particular way by humans to achieve specified purposes. The resistor serves a particular purpose in each human construct in which it is found, a purpose that draws its meaning from human activities and values. It is more illuminating to look first at the purpose of missiles or heart-lung machines, and then to look at how the resistor helps to achieve this purpose.

This idea then leads to the second approach (the one I prefer) to the significance of values in game theory mathematics. This is to look at the purposes of the mathematics (and the values associated with these purposes) and then to see if the mathematics is congruent with these purposes. If there is a reasonably close relationship between the form and organization of the mathematics and the concepts used to interpret it, then it makes sense to call the mathematics 'biased in itself'.

Take again equation (*). It is easy to associate the discrete number x with the existence of separate and fixed players, the separate terms (such as $3f_1$ and $-2f_2$) with the existence of discrete and fixed choices, and the coefficients (3, -2) with possible payoffs. Furthermore, the solution to the game — the fixed values for f_1 and f_2, and a numerical value for x — reflects the rooting of game theory formulations in a pre-selected set of possible actions and results. In these and other ways there is a clear relationship between the concepts of game theory and the mathematics that is associated with the ideas.

6. INTERPRETING GAME THEORY

To be really confident in tracing the selective usefulness of a theory to values in its fundamental concepts, it is probably necessary to be able to point to an alternative theory, with different in-built values and a different selective usefulness. For game theory such an alternative appears not to be available. (Game theory itself is an alternative to certain models of economics, but reflects many of the same values as these models.) It is worthwhile to remember that the large majority of mathematicians and scientists tend to think in terms of the prevailing concepts in society and are stimulated by problems generated by social institutions as they exist, existed, or are seen to exist. Since von Neumann was one of the top mathematicians of the twentieth century, it is not surprising that no alternative has been posed. Even if it had, it would have been very unlikely to receive the same concentrated attention (by intellectuals in government and military bureaucracies in particular) that fell to game theory.

In addition, there is no guarantee that a *mathematical* theory with values radically different than those of game theory could really be feasible. Consider the possibility of a theory based on modelling small groups of people managing their lives and their local environment, basing their decisions on collective evaluations based on equity, developing the potentials of individuals, and so on — in other words a cooperative, collective, non-hierarchical social structure. A mathematical theory might be able to model the sorts of interactions in such a society. But would anyone think it worth modelling, if the decisions always came out of collective discussion, give-and-take, and study of alternatives, and not out of a model?

In some areas of mathematics alternative theories exist — for example, catastrophe theory in biology[37] — and can throw great light on the origin of the selective usefulness of different mathematical formalisms. This is not so in the case of game theory. We apparently must be content to look at the values associated with the concepts and their related mathematical formalism, and at the actual applications and uses of the theory.

The idea presented here of value-laden concepts and selective usefulness of a mathematical formalism is not unusual in terms of other available critiques or of the perspective of the sociology of knowledge, but is different from conventional interpretations of game theory. One way of looking at the applications of game theory is in terms of a use-abuse framework.[38] Many defenders of game theory would say that many or most of its applications (and certainly its ex post facto use as a legitimization of policies) in practice constitute 'misuses' of the theory: the theory was not designed for these purposes. These defenders can go to great lengths to spell out when game theory is appropriate and when it is not. But from the point of view of game theory's value-laden concepts and selective usefulness, these 'misuses' are not due so much to fault of users as to the bias in the theory itself: the theory *lends* itself to 'misuse'. Everyone knows that one shouldn't use automobiles to kill people, but this seems to be an unavoidable consequence of using them as a form of transport. It *is* possible to trace social consequences to 'inappropriate technology', or to an 'inappropriate mathematical framework'.

Another, more sophisticated approach to the uses of game theory is to say that game theory may not be very good in arriving at precise strategies in complex situations, but that it is useful in helping one to think about the situation in an ordered way.[39] In this view, game theory is valuable as a tool of conceptual analysis rather than as a direct mathematical tool (for the situations which are 'misuses' in the previous approach). But of course from the point of view of value-laden concepts, this is precisely what game theory is not useful for. By using the concepts of game theory to help oneself think about a situation, it is difficult indeed not to be led to think primarily in terms of those concepts, and unconsciously to incorporate their values. Indeed, one of the things that makes this analysis of game theory hard for me is the perpetual inclination to think in terms of a game theory formulation.

• *NOTES*

I would like to thank Henri de Feraudy, Alec McHoul, Thomas Mautner, Martin Ward, David Edge and two anonymous referees for this journal, for valuable comments.

1. B. Hessen, 'The Social and Economic Roots of Newton's "Principia"', in

N. Bukharin et al., *Science at the Cross Roads* (London: Kniga, 1931, reprinted by Frank Cass & Co. Ltd., 1971), 151-212 details the influences of bourgeois capitalist social and economic structures on the work of Isaac Newton; P. Forman, 'Weimar Culture, Causality, and Quantum Theory, 1918-1927: Adaptation by German Physicists and Mathematicians to a Hostile Intellectual Environment', *Historical Studies in the Physical Sciences,* Vol. 3 (1971), 1-115 describes the influences of post-World War I German culture on the rise of quantum mechanics; R. Cowan, 'Francis Galton's Statistical Ideas: the Influence of Eugenics', *Isis,* Vol. 63 (1972), 509-28 describes the influence of Francis Galton's ideas about eugenics on his statistical ideas; R. Young, 'The Historiographic and Ideological Contexts of the Nineteenth-Century Debate on Man's Place in Nature', in M. Teich and R. Young (eds), *Changing Perspectives in the History of Science: Essays in Honour of Joseph Needham* (London: Heinemann, 1973), 344-438 documents the influence of the ideas of Malthus and others concerning the structure of society upon the concepts of Darwinian evolution: R. Hall, *Food for Nought: The Decline in Nutrition* (New York: Harper & Row, 1974), 119 comments on the chemical industry's influence on academics' ideas on controlling insects. A good survey of these and other 'external' influences on scientific knowledge is given by B. Barnes, *Scientific Knowledge and Sociological Theory* (London: Routledge & Kegan Paul, 1974). See also the papers by Bernard Norton and Donald MacKenzie in this issue of *Social Studies of Science,* on the work of Karl Pearson.

2. For example, J. Miller, 'The Dog Beneath the Skin', *The Listener,* Vol. 88 (20 July 1972), 74-76 tells how a physiological model of the nervous system (taken from an evolutionary model of mob society) was used 'explain' mob psychology. See also many of the essays by Charles Rosenberg in his *No Other Gods* (Baltimore, Md., and London: The Johns Hopkins University Press, 1976). Some of the references in note 1 describe or suggest the selective usefulness of the knowledge achieved under 'outside' influences. The selective practical usefulness of much scientific knowledge — such as knowledge produced as a result of much military or industrial research — is not hard to recognize.

3. Young, op. cit. note 1; R. Hofstadter, *Social Darwinism in America* (Boston: Beacon Press, 1955).

4. Cowan, op. cit. note 1.

5. Forman, op. cit. note 1.

6. T. Kuhn, *The Structure of Scientific Revolutions* (Chicago: The University of Chicago Press, 1970).

7. P. Feyerabend, *Against Method: Outline of an Anarchistic Theory of Knowledge* (London: New Left Books, 1975).

8. The use/abuse framework might still be considered to apply even if tools are value-laden, since before a tool can have fortunate or unfortunate consequences it must be 'used'. This seems to me to remove somewhat the ideas of 'use' and 'abuse' from their normal reference framework. Typically tools are designed for a certain purpose, and one 'uses' them for that purpose or 'abuses' them for an altogether different purpose. Many of the cases of the value-ladenness of tools may be better likened to design problems: the tool does not achieve what it (allegedly) was designed to do. The case of the stereo amplifier is an example. (The use/abuse framework may still be retained, in a tautological way, by saying that what a tool is actually useful for defines what it 'should' be used for.) More fundamentally, the appropriateness or inappropriateness of a tool always depends on the wider context

in which it is used — the prevailing historical, social, and other circumstances. So it is possible that tools designed for one purpose may sometimes be used for other purposes. Even anti-personnel weapons may eventually have some different, beneficial application. (Obviously 'beneficial' here refers to the values of the user — whether those of American militarists or Vietnamese peasants — which in turn reflect one's position in society culturally, ethically, and so forth.) Since we do not know how game theory may prove useful in the different circumstances that will prevail in the future, there is no guaranteed way of knowing what values are 'ultimately' intrinsic to the theory. However, this observation does not negate the idea that values may be intrinsic to game theory, but reflects the contingency of values themselves on historical, social, and other circumstances. I certainly believe that the question of whether values are ultimately intrinsic to game theory is worth further, more detailed attention (although not in this present paper). In any case, the present analysis can always be qualified as being concerned only with the values embedded in game theory in the current historical era.

For a thorough discussion and application of the metaphor of 'ideas as tools', which underlies this paper, see Barnes, op. cit. note 1.

9. The best account of game theory which combines accessibility with comprehensiveness and a reasonable amount of rigour is R. Luce and H. Raiffa, *Games and Decisions: Introduction and Critical Survey* (New York: John Wiley, 1957). An excellent general introduction is A. Rapoport, *Fights, Games, and Debates* (Ann Arbor, Mich.: University of Michigan Press, 1960). For a concise but incisive account, see A. Tucker and H. Kuhn, 'Games, Theory of', *Encyclopaedia Britannica,* Vol. 9 (1969), 1121-26. M. Shubik (ed.), *Game Theory and Related Approaches to Social Behavior* (New York: John Wiley, 1964) provides a good taste of related ideas and applications. For the mathematically oriented, the classic treatment is J. von Neumann and O. Morgenstern, *Theory of Games and Economic Behavior* (Princeton, NJ: Princeton University Press, 1943).

10. A. Rapoport, *Strategy and Conscience* (New York: Harper & Row, 1964), Chapter 11.

11. 'A perusal of the literature on applications of game theory to logistic and military problems reveals that *all* of them are cast in the form of two-person zero-sum games.' A. Rapoport, 'Prisoner's Dilemma — Recollections and Observations', in Rapoport (ed.), *Game Theory as a Theory of Conflict Resolution* (Dordrecht: D. Reidel, 1974), 33.

12. In a formal game theory analysis (with the game in normal form), each player chooses a single 'strategy' (a decision which covers specific responses to all possible move sequences by other players) rather than making a succession of choices. The term 'choice' is used here for simplicity; in any case the associated values are virtually identical.

13. For a discussion of the military applications of game theory, see for example J. Grote (ed.), *The Theory and Application of Differential Games* (Dordrecht: D. Reidel, 1974), and A. Mensch (ed.), *Theory of Games: Techniques and Applications* (London: The English Universities Press, 1966).

14. R. Isaacs, 'The Past and Some Bits of the Future', in Grote, op. cit. note 13, 1-11.

15. A. Blanquière, F. Gérard and G. Leitmann, *Quantitative and Qualitative Games* (New York: Academic Press, 1969) is an example of pure mathematics done using game theory as a takeoff point. Rapoport, op. cit. note 11, comments that the

benefits of this sort of intellectual effort go to professional practitioners, not to humanity.

16. P. Green, *Deadly Logic: The Theory of Nuclear Deterrence* (Columbus, Ohio: Ohio State University Press, 1966).

17. I. Horowitz, 'Deterrence Games: from Academic Casebook to Military Codebook', in P. Swingle (ed.), *The Structure of Conflict* (New York: Academic Press, 1970), 277-96.

18. It might be thought that the ostensible reason for using game theory is the justification of preconceived conclusions as such. While not arguing with such a formulation, I would not wish to imply that there was ever any such conscious intention by the users.

19. See, for example, the applications to environmental decision-making, A. Ostrom, 'A Review of Conflict Resolution Models in Water Resources Management', in A. Szöllösi-Nagi (ed.), *Workshop on the Vistula and Tisza River Basins, 11-13 February 1975* (Laxenburg: International Institute for Applied Systems Analysis, 1976), 96-105; and to cultural values, W. Goldschmidt, 'Game Theory, Cultural Values, and the Brideprice in Africa', in I. Buchler and H. Nutini (eds), *Game Theory in the Behavioral Sciences* (Pittsburgh, Penn.: University of Pittsburgh Press, 1969), 61-74.

It is only fair to note that there are some applications of game theory inspired by awareness of the limitations of the theory in particular contexts. For example, A. Rapoport (see especially op. cit. note 10) strongly criticizes many of the applications of game theory to international strategic situations, and for example uses the prisoner's dilemma game to illustrate the irrationality of non-cooperation in certain situations. However, it is not game theory but Rapoport (and his readers) that provides the insight. The prisoner's dilemma game is the prime example of a case where theorists have struggled mightily for literally decades, producing amazing mathematical and logical constructions, all in an attempt to find a way for the theory to give conclusions congruent with commonsense — and with a lack of any real success. The selective usefulness of game theory concepts makes it difficult indeed to model cooperative behaviour in a non-trivial way. So while students of conflict no doubt have gained insights while using game theory, they might well have achieved much greater progress using a different conceptual framework.

20. G. Snyder, *Deterrence and Defense: Toward a Theory of National Security* (Princeton, NJ: Princeton University Press, 1961), 270.

21. See, for example, the historical studies by D. Fleming, *The Cold War and its Origins, 1917-1960* (London: Allen & Unwin, 1961), and D. Horowitz, *From Yalta to Vietnam: American Foreign Policy in the Cold War* (Harmondsworth, Middx.: Penguin Books, 1967).

22. A critical study of this example is found in Rapoport, op. cit. note 10, 92-93.

23. It is questionable how much the typical strategists who model international relations actually apply concepts from game theory, and how much they apply a distorted version of game theory. For example, Snyder's example (op. cit. note 20) is not a proper game theory formulation. Thus it might be said that many such applications are 'misuses' of the theory. Even so, the use of game theory as a conceptual inspiration in strategic studies of international relations still reflects the selective usefulness of game theory concepts for particular types of purposes. For it remains the case that the strategists chose to use game theory concepts for their purposes (rather than adopting concepts from other theories, or creating their

own framework), and also without much trouble ended up justifying their conclusions thereby. So it might be said that the theory 'lends itself to misuse', a phrase which tends to shift the blame away from the user.

24. R. Braithwaite, *Theory of Games as a Tool for the Moral Philosopher* (Cambridge: Cambridge University Press, 1955).

25. M. Willmer, *Crime and Information Theory* (Edinburgh: Edinburgh University Press, 1970).

26. E. Sutherland, *White Collar Crime* (New York: Dryden Press, 1949).

27. N. Morris and G. Hawkins, *The Honest Politician's Guide to Crime Control* (Chicago: The University of Chicago Press, 1970).

28. Von Neumann's proof of the fundamental 'minimax theorem' dates from 1928, but I do not treat it here independently of his later work with Morgenstern. Emile Borel, another mathematician, whose work in the 1920s presaged, or initiated (depending on one's viewpoint) game theory, was from the beginning aware of its implications for the study of military, economic, and financial problems, as well as the direct link with games of chance and skill. See M. Fréchet, 'Emile Borel, Initiator of the Theory of Psychological Games and its Application', *Econometrica*, Vol. 21 (1953), 95-127.

29. Von Neumann and Morgenstern, op. cit. note 9.

30. Ibid., 1.

31. A referee has remarked that my case 'could be strengthened by the observation that the model of the actor and of action on which game theory is built is taken (as the author in effect demonstrates) from that prevalent both in theory (economic man, etc.) and to some extent in practice, in capitalist societies . . . . Some of the analyses of Lukacs in his *History and Class Consciousness* have relevance here.' I am sympathetic with this remark, but have reservations. For while the basic model of game theory may be taken from the theory and practice of economic man (etc.), this is only obvious in retrospect (by looking at game theory and at economic man). Also, I would prefer not to suggest any conscious intent to use prevailing cultural models in designing game theory.

32. O. Morgenstern, *The Question of National Defense* (New York: Random House, 1959).

33. Note, for example, funding by NATO and the organizational locations of contributors in Mensch, op. cit. note 13; H. Kuhn and G. Szegö (eds), *Differential Games and Related Topics* (Amsterdam: North-Holland, 1971); Grote, op. cit. note 13.

34. Isaacs, op. cit. note 14.

35. A referee, in declaring that this is 'the least convincing section of the paper', complains at the example I have chosen. He argues that I take 'a pre-existing bit of mathematical theory (the theory of systems of linear equations) and can only reach a very weak conclusion as to the links between it and the origins, etc., of game theory. A more promising approach would surely be to take new areas of mathematics opened up by game theorists and to attempt a similar analysis on them.'

My response to this comment is two-fold. First, the purpose of this simple example is to present a point of view, not to prove the existence of links. A simple example is necessary for non-mathematicians. The example of simultaneous linear equations is almost too trivial to count as game theory mathematics. It has only been used to illustrate the ways of looking at links between concepts and mathematics. Such links will be found, if at a more recondite level, throughout game theory. Second, it seems to me that most of the simple game theory mathematics (that used in actual applications) can be translated into old, 'standard' mathematics. But much of the importance of game theory involves the particular concepts attached to the mathematics, the form of the (new) theorems proved, and so on. What is 'new' mathematics, anyway? Much of it is old mathematics dressed up in new garbs (e.g. 'solving a new problem with old tools'). Part of the problem here is that new, 'applied' mathematics (of which game theory is for the most part an example) often uses quite a bit of old, 'pure' mathematics. It would take considerable effort to carry the analysis deep into pure mathematics. This would be a worthwhile task (though even then I would be skeptical about convincing pure mathematicians), but I am not convinced that this present paper is the place to do it.

36. Of course these links are seldom direct; typical mediating factors are outstanding scientific problems (themselves perhaps influenced by commercial or political factors) and beliefs about the structure of thought. The general relation between 'pure' mathematics and 'reality' may be stated as follows:

> The history of mathematics reveals that the interest in the formal processes of mathematics was seldom divested of the desire to obtain an adequate picture of the physical universe. The postulates and operations of analysis are not chosen *arbitrarily,* but are postulates and operations which map the geometrical order of things in the abstract realm of numbers. The formal operations of analysis are thus merely one link in our desire to discover the inherent functional order of the physical universe. (C. Lanczos, *Applied Analysis* [London: Pitman & Sons, 1957], 1)

If it is accepted that the perceived nature of the 'physical universe' depends at least partially on human interests (for example via the study of ballistics, gambling, or navigation), and remembering that mathematics is used to model social, political, and economic universes, then it becomes possible to see the extent of the 'impact of society on mathematics'. Some of the attempts to spell out the links between the form and content of mathematics and structures of society are: E. Burtt, *In Search of Philosophic Understanding* (London: Allen & Unwin, 1967), 163; M. Thomas, 'The Faith of the Mathematician', in T. Pateman (ed.), *Counter Course: a Handbook for Course Criticism* (Harmondsworth, Middx.: Penguin Books, 1972), 187-201; A. Sohn-Rethel, 'Mental and Manual Labour in Marxism', in P. Walton and S. Hall (eds), *Situating Marx: Evaluations and Departures* (London: Human Context Books, n.d.), 44-71; L. Hodgkin, 'Politics and Physical Science', *Radical Science Journal,* No. 4 (1976), 29-60.

37. R. Thom, *Structural Stability and Morphogenesis: an Outline of a General Theory of Models* (Reading, Mass.: W.A. Benjamin, 1975).

38. 'No theory is moral and no theory is immoral. The question of morality does not apply, it just does not arise.' O. Morgenstern, 'On Some Criticisms of Game Theory', in Mensch, op. cit. note 13, 450. See also note 8.

39. The value of game theory as a tool for description, as a meaningful frame of reference, is claimed by A. Rapoport, 'Conflict Resolution in the Light of Game Theory and Beyond', in Swingle, op. cit. note 17, 32; R. Singleton and W. Tyndall,

Games and Programs: Mathematics for Modeling (San Francisco: Freeman, 1974); and many others.

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