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Magneto-optical effects in magnetic white dwarfs – II. Continuum

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Summary. The impact on the spectra of magnetic white dwarfs of magneto-optical effects or magnetic birefringence arising from free electrons in the continuum is investigated using model calculations. Linear polarization is the most strongly affected: in the continuum its magnitude is reduced and its sign often reversed, and in the lines its magnitude is greatly reduced. In the outer components of lines, the maximum values of circular polarization are reduced and their positions shifted inwards. Line depths are increased, especially the central component of a Zeeman triplet.

1 Introduction

About a dozen white dwarf stars exhibit spectra suggesting the presence of high ($\gtrsim 3$ MG) magnetic fields (Angel 1978). The suggestive features are either significant values of circular polarization, or the shifting and splitting of absorption lines (Zeeman splitting). Early attempts to explain these features relied on approximate calculations (see e.g. Angel 1977). In a series of papers we have attempted to model these stars in detail using realistic stellar atmospheres and full integration of the radiative transfer equations through the appropriate optical depths and across the surface of the star using realistic line profiles (Martin & Wickramasinghe 1978; Wickramasinghe & Martin 1979; see also O'Donoghue 1980). The key unknowns estimated by comparing model results with observed spectra are the magnitude of the star's magnetic field, which influences especially the positions of absorption features, and the angle between the magnetic field axis and the viewing direction, which influences especially the magnitude of circular polarization.

There are quite a number of unprobed assumptions involved in these calculations, particularly that the magnetic field configuration is that of a centred dipole and that the magnetic field has no significant effects on the temperature and pressure structure of the atmosphere, and a number of approximations, including numerical interpolations and

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solutions and the determination of polarization of the continuum. To determine whether the spectra of magnetic stars can be fully explained under the assumptions adopted, it is necessary to consider all physical phenomena which may significantly affect the spectrum. In Martin & Wickramasinghe (1981), henceforth denoted Paper I, we considered one such effect, the impact of magneto-optical effects associated with spectral lines. (Magneto-optical effects, also known as magnetic birefringence or anomalous dispersion, are in special cases known as Faraday rotation and the Voigt effect: see Paper I.) In this paper we extend that analysis to consider magneto-optical effects arising from free electrons in the continuum.

To describe the transfer of polarized light in the presence of a magnetic field, it is usual to write equations in terms of the four Stokes parameters I, Q, U and V (Hardorp, Shore & Wittmann 1976):

$$\mu \frac{dI}{d\tau} = \eta_I(I-B) + \eta_Q Q + \eta_V V, \tag{1}$$

$$\mu \frac{dQ}{d\tau} = \eta_Q(I-B) + \eta_I Q - \rho_R U, \tag{2}$$

$$\mu \frac{dU}{d\tau} = \rho_R Q + \eta_I U - \rho_W V, \tag{3}$$

$$\mu \frac{dV}{d\tau} = \eta_V(I-B) + \rho_W U + \eta_I V, \tag{4}$$

where τ is the optical depth, $\mu = \cos\theta$ where θ is the angle between the propagation direction and the axis along which τ is measured, B is the local source function and

$$\eta_I = \tfrac{1}{2}\eta_p \sin^2\psi + \tfrac{1}{4}(\eta_l + \eta_r)(1 + \cos^2\psi), \tag{5}$$

$$\eta_Q = [\tfrac{1}{2}\eta_p - \tfrac{1}{4}(\eta_l + \eta_r)] \sin^2\psi, \tag{6}$$

$$\eta_V = \tfrac{1}{2}(\eta_r - \eta_l)\cos\psi, \tag{7}$$

where ψ is the angle between the direction of propagation and the direction of the magnetic field and η_p, η_l and η_r are the ratios of the total absorption coefficients (κ_p, κ_l, κ_r) of the three shifted Zeeman components plus the shifted continuum absorption coefficients (κ_{cp}, κ_{cl}, κ_{cr}) to the unshifted continuum absorption coefficient ($\kappa_c = \kappa_{cp}$). The solution pair

$$\begin{pmatrix} Q \\ U \end{pmatrix}$$

to (1)–(4) must be multiplied by

$$\begin{pmatrix} \cos 2\phi & -\sin 2\phi \\ \sin 2\phi & \cos 2\phi \end{pmatrix}$$

where ϕ is the azimuth with respect to an arbitrary x-axis at right angles to the line-of-sight.

When there is no magnetic field, $\eta_r = \eta_l = \eta_p$, hence $\eta_Q = \eta_V = 0$ and the radiative transfer is described by a single equation. In the presence of a magnetic field, η_r and η_l are different from η_p, hence η_Q and η_V are non-zero, and the system (1)–(4) leads to Zeeman splitting in the intensity I of the absorption lines, circular polarization V/I and linear polarization of magnitude $(Q^2 + U^2)^{1/2}/I$ orientated at an angle ½ $\tan^{-1}(U/Q)$ with respect to the polar axis of the star.

The parameters ρ_R and ρ_W introduce magneto-optical effects: ρ_R leads to a rotation of the electric vector of linearly polarized light, while ρ_W leads to a phase retardation between the linear polarizations which are parallel to and perpendicular to the magnetic field. In Paper I we investigated the impact of magneto-optical effects associated with line absorption. The main effect of ρ_R and ρ_W in this case is to deepen the central component of an overlapping Zeeman triplet and to introduce anti-symmetric bulges in circular polarization near the central component. In this paper we analyse the effect of contributions to ρ_R and ρ_W arising from free electrons in the continuum.

A number of authors (e.g. Sazonov & Chernomordik 1975; Angel 1977) have argued that because the plane of polarization of light emitted from different optical depths and different points on the surface of a magnetic white dwarf is rotated by different amounts due to magneto-optical effects, linear polarization, which is an average over the star's surface, will be greatly reduced. Landstreet (1980) argues to the contrary that magneto-optical rotation affects primarily that mode of polarization which is not emitted anyway (such as linear polarization at $\psi = 0$) and hence the emergent spectrum is essentially unaffected by the magneto-optical effects.

Our results do show a linear depolarization effect from the magneto-optical effects due to free electrons, in agreement with the first of the above views. Yet the situation is not as simple as the idea of 'depolarization' suggests, nor are Landstreet's considerations entirely misplaced.

Landstreet is correct, as will be seen later, in his reasoning and conclusion that in the special cases in which the propagation direction is parallel or perpendicular to the field lines ($\cos\psi = 1$ or 0), the magneto-optical effects have no effect on the emergent spectrum. Even at oblique angles ($\cos\psi \neq 1$ or 0) the values of the magneto-optical parameters may be such as to preclude any significant effect on the spectrum, as in the case of the outer components of a Zeeman triplet when only ρ_R and ρ_W arising from the lines are considered (Paper I).

When magneto-optical effects arising from the continuum are considered at oblique angles, there can be a significant reduction in linear polarization as suggested by a number of authors and contrary to Landstreet's view. But most authors arguing the case for depolarization have considered it to be virtually complete depolarization, whereas the contributions near $\cos\psi = 1$ or 0 where depolarization does not occur mean that depolarization, when averaged over the surface of the star, can at most be partial. Another effect overlooked by most authors is *re*polarization. Large values of ρ_R and ρ_W at oblique angles do indeed obliterate linear polarization which would otherwise be roughly proportional to η_Q. But the magneto-optical parameters also can create linear polarization of similar magnitude as a by-product of circular polarization. Prior authors also have not mentioned that magneto-optical effects can lead to significant alterations in values of circular polarization and flux in the lines when the propagation direction and field direction are oblique. Indeed, these effects would be very pronounced were it not for contributions near $\cos\psi = 1$ and 0 which reduce the net effect.

It is not surprising that magneto-optical effects on flux and circular polarization, and the phenomenon of repolarization, have not been widely predicted, since the physical processes involved are complex and hard to conceptualize. The solution to (1)–(4) must be averaged over the surface of a star, with all the different values of opacities, angles and magneto-optical parameters involved, and then considered at different wavelengths. This difficulty suggests the importance of informing physical insight with an understanding of mathematical solutions.

2 Theory

Contributions to the magneto-optical parameters ρ_R and ρ_W due to free electrons may be written (Pacholczyk 1976, pp. 99, 103)

$$\rho_R = -\frac{\omega_0^2 \omega_G \cos\psi}{c\kappa_c(\omega^2 - \omega_G^2)}, \tag{8}$$

$$\rho_W = -\frac{\omega_0^2 \omega_G^2 \sin^2\psi}{2c\omega\kappa_c(\omega^2 - \omega_G^2)}, \tag{9}$$

where ω is the angular frequency of the radiation, $\omega_0 = (4\pi \mathrm{Ne}^2/m_e)^{1/2}$ is the plasma frequency, $\omega_G = eB/m_e c$ is the cyclotron frequency and κ_c is the unshifted continuum absorption coefficient per unit volume. These expressions have to be modified as the cyclotron frequency is approached, but in the present paper we assume $\omega \gg \omega_G$. Note that both ρ_R and η_V are proportional to $\cos\psi$ while both ρ_W and η_Q are proportonal to $\sin^2\psi$; this is also the case for contributions to ρ_R and ρ_W from the lines (Paper I), which should be added to the values (8) and (9) along with any further contributions from other sources.

In the continuum, where other contributions to ρ_R and ρ_W are negligible, $\rho_R/\rho_W = 2\omega\cos\psi/(\omega_G \sin^2\psi)$. For a typical field strength $B = 10^7$ G for a magnetic white dwarf, $2\omega/\omega_G = 42.8$ at $\lambda = 5000$ Å, so ρ_R will usually be much larger than ρ_W in the continuum. (In the continuum, linear polarization is normally much smaller than circular polarization. The continuum opacities κ_{cr} and κ_{cl} are determined primarily by a simple shift of the wavelength-dependent absorption coefficient (Lamb & Sutherland 1974). Since κ_c is smooth and approximately linear for variations in ω of the order ω_G, $\kappa_{cp} - \frac{1}{2}(\kappa_{cr} + \kappa_{cl}) \ll \kappa_{cp}$, hence $\eta_Q \ll \eta_V$ and $(Q^2 + U^2)^{1/2} \ll V$.)

The impact of the free-electron magneto-optical effects on the solutions to the radiative transfer equations (1)–(4) can be determined readily numerically (Martin & Wickramasinghe 1979). But to illustrate the influence of the magneto-optical effects and provide some mathematical and physical insight, it is convenient to refer to the solution to the radiative transfer equations for an Unno atmosphere with source function linear in the optical depth: $B = B_0(1 + \beta\tau)$. When $\rho_R = \rho_W = 0$, the Unno solution (Unno 1956), called here Unno solution 1, is

$$I_1 = B_0(1 + \beta\mu\eta_I/D_1), \tag{10}$$

$$Q_1 = -B_0\beta\mu\eta_Q/D_1, \tag{11}$$

$$U_1 = 0, \tag{12}$$

$$V_1 = -B_0\beta\mu\eta_V/D_1, \tag{13}$$

where

$$D_1 = \eta_I^2 - \eta_Q^2 - \eta_V^2. \tag{14}$$

When $\rho_R \neq 0$, $\rho_W \neq 0$, the Unno solution to (1)–(4) may be written (Unno solution 2)

$$I_2 = B_0(1 + \beta\mu\eta_I/D_2), \tag{15}$$

$$Q_2 = -\left(\frac{1 + \rho_R\rho_W/(\eta_I^2 + \rho_W^2)(\eta_V/\eta_Q)}{1 + \rho_R^2/(\eta_I^2 + \rho_W^2)}\right) B_0\beta\mu\eta_Q/D_2, \tag{16}$$

$$U_2 = -\left(\frac{\rho_W \eta_V - \rho_R \eta_Q}{\eta_I^2 + \rho_R^2 + \rho_W^2}\right) B_0 \beta \mu \eta_I / D_2, \tag{17}$$

$$V_2 = -\left(\frac{1 + \rho_R \rho_W/(\eta_I^2 + \rho_R^2)(\eta_Q/\eta_V)}{1 + \rho_W^2/(\eta_I^2 + \rho_R^2)}\right) B_0 \beta \mu \eta_V / D_2, \tag{18}$$

where

$$D_2 = \eta_I^2 - \left(1 - \frac{\rho_R^2}{\eta_I^2 + \rho_R^2 + \rho_W^2}\right)\eta_Q^2 - \frac{2\rho_R \rho_W \eta_Q \eta_V}{\eta_I^2 + \rho_R^2 + \rho_W^2} - \left(1 - \frac{\rho_W^2}{\eta_I^2 + \rho_R^2 + \rho_W^2}\right)\eta_V^2. \tag{19}$$

The solution (15)–(19) has been expressed in a manner analogous in form to (10)–(14). When $\rho_R = \rho_W = 0$ it is apparent that Unno solution 2 reduces to Unno solution 1, and also that ρ_R and ρ_W, which according to (1)–(4) lead to mixing between Q, U and V, have a direct effect on Q_2, U_2 and V_2 and an indirect effect, via D_2, on I_2.

Consider the above solutions in the special case in which the propagation direction is parallel to the magnetic field: $\cos\psi = 1$. Then $\eta_I = ½(\eta_r + \eta_l)$, $\eta_Q = 0$ and $\eta_V = ½(\eta_r - \eta_l)$. (It is clear in this case that the circular polarization V/I, proportional to η_V, is the difference between the right and left circularly polarized fractions.) Equations (10)–(14) become Unno solution 3:

$$I_3 = B_0(1 + B\mu\eta_I/D_3), \tag{20}$$

$$Q_3 = 0, \tag{21}$$

$$U_3 = 0, \tag{22}$$

$$V_3 = -B_0\beta\mu\eta_V/D_3, \tag{23}$$

$$D_3 = \eta_r\eta_l. \tag{24}$$

Because in this case $\rho_W = 0$, this solution also holds when magneto-optical effects are included (equations 15–19 when $\cos\psi = 1$). As noted by Landstreet (1980), the only non-zero magneto-optical parameter ρ_R merely mixes contributions to Q and U, which do not appear anyway. When propagation is perpendicular to the magnetic field lines ($\cos\psi = 0$), then $\eta_I = ½\eta_p + ¼(\eta_r + \eta_l)$, $\eta_Q = ½\eta_p - ¼(\eta_r + \eta_l)$, $\eta_V = 0$ and (10)–(14) become Unno solution 4:

$$I_4 = B_0(1 + \beta\mu\eta_I/D_4), \tag{25}$$

$$Q_4 = -B_0\beta\mu\eta_Q/D_4, \tag{26}$$

$$U_4 = 0, \tag{27}$$

$$V_4 = 0, \tag{28}$$

$$D_4 = ½\eta_p(\eta_r + \eta_l). \tag{29}$$

Again, as noted by Landstreet (1980), the same solution holds when magneto-optical effects are included, since $\rho_R = 0$.

The nature of the solutions presented so far is shown in Figs 1, 2 and 3, for which $\cos\psi = 1$, 0.7 and 0 respectively. Intensity, linear and circular polarization are shown in each figure across an idealized Zeeman triplet calculated using Unno solution 1, Unno solution 2 with ρ_R and ρ_W from lines only, and Unno solution 2 with ρ_R and ρ_W from lines and continuum. In Figs 1 and 3 these three solutions are identical, and are the Unno solutions

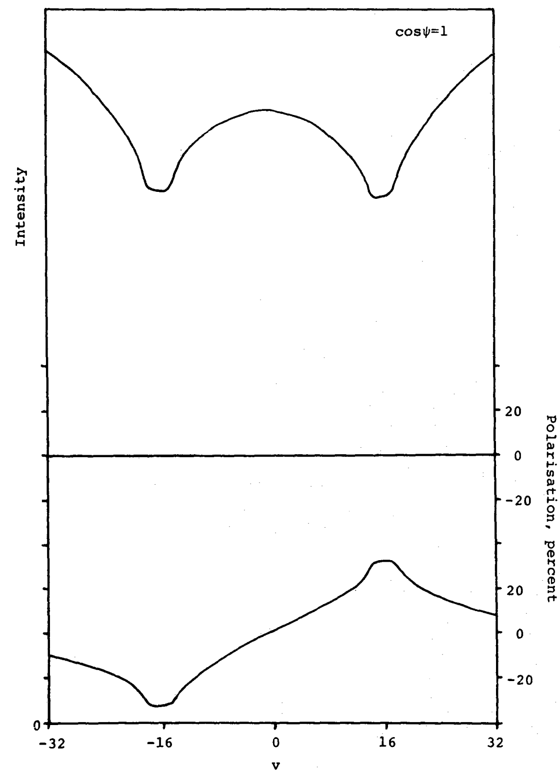

Figure 1. Intensity (top), linear polarization (middle) and circular polarization (bottom) for a hypothetical Zeeman triplet. A Voigt line profile is adopted (see Paper I). The position within the profile is given by $v = \Delta\lambda/\Delta\lambda_D$ where $\Delta\lambda_D$ is the Doppler width, and $v_r = -16$, $v_p = 0$, $v_l = 16$, $a = 0.1$, $\mu = 0.8$, $\cos\psi = 1$ and $\cos 2\phi = 0.6$. The continuum opacities are set at $\eta_p = 1.00$, $\eta_l = 0.93$ and $\eta_r = 1.08$, while the line depth is set at $\eta_0 = 10\,000$ when $a = v = 0$. Values of ρ_R and ρ_W associated with the lines are taken from Paper I, and values from free electrons are set at $\rho_R = -20\,000 \cos\psi$ and $\rho_W = -400 \sin^2\psi$. The values of η_p, η_l, η_r, ρ_R and ρ_W are typical for the atmosphere adopted, which is the zero-field, high-gravity ($\log g = 8.0$) model atmosphere for a magnetic white dwarf with $T_e = 20\,000$ K given by Wickramasinghe (1972), with $\lambda(v = 0) = 5000$ Å. The solutions are obtained using algorithms in Martin & Wickramasinghe (1979), and for this case in which the field lines are parallel to the propagation direction ($\cos\psi = 1$) the solutions are unchanged when magneto-optical parameters are included. The solution is qualitatively similar to Unno solution 3, equations (20)–(24).

3 and 4 respectively. This is the situation described by Landstreet (1980). Fig. 2 shows the dramatic impact of magneto-optical effects from free electrons on linear polarization which occurs at oblique angles, predicted by Sazonov & Chernomordik (1975), Angel (1977) and others. Also apparent are significant changes in circular polarization and in flux, especially in the outer components. The contrasting types of results in Figs 1 to 3 all play a role when a solution averaged over the surface of a star is calculated.

Next consider the case in which ρ_R and ρ_W are much greater than η_I, η_Q or η_V, which will normally be the case in the continuum and the wings of lines. The solution (15)–(19) becomes approximately (Unno solution 5).

$$I_5 = B_0(1 + \beta\mu\eta_I/D_5), \tag{30}$$

$$Q_5 = -\left(\frac{\rho_W^2 + \rho_R\rho_W\,\eta_V/\eta_Q}{\rho_R^2 + \rho_W^2}\right) B_0\beta\mu\eta_Q/D_5, \tag{31}$$

$$U_5 = -\left(\frac{\rho_W\eta_V - \rho_R\eta_Q}{\rho_R^2 + \rho_W^2}\right) B_0\beta\mu\eta_I/D_5, \tag{32}$$

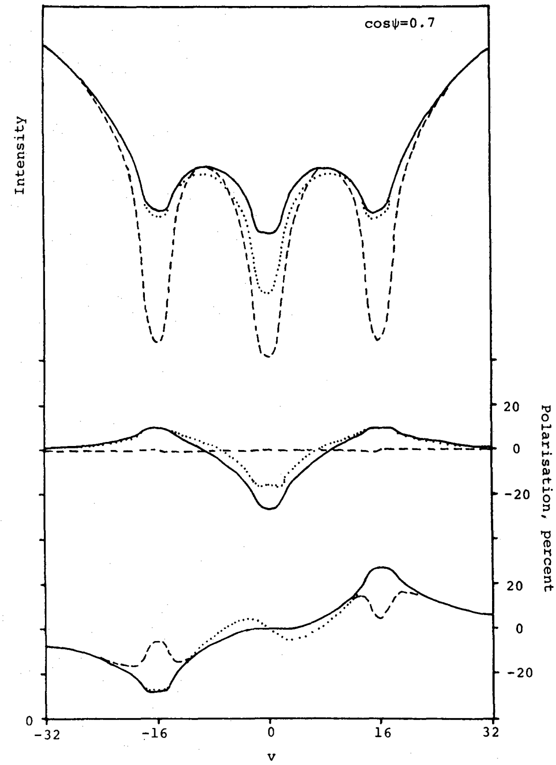

Figure 2. As Fig. 1, but with $\cos\psi = 0.7$. The solid line is the solution in which $\rho_R = \rho_W = 0$, the dotted line is the solution with ρ_R and ρ_W contributions from the lines only, and the dashed line is the solution with ρ_R and ρ_W contributions from lines and from free electrons in the continuum. The solution is qualitatively similar to Unno solution 2, equations (15)–(19).

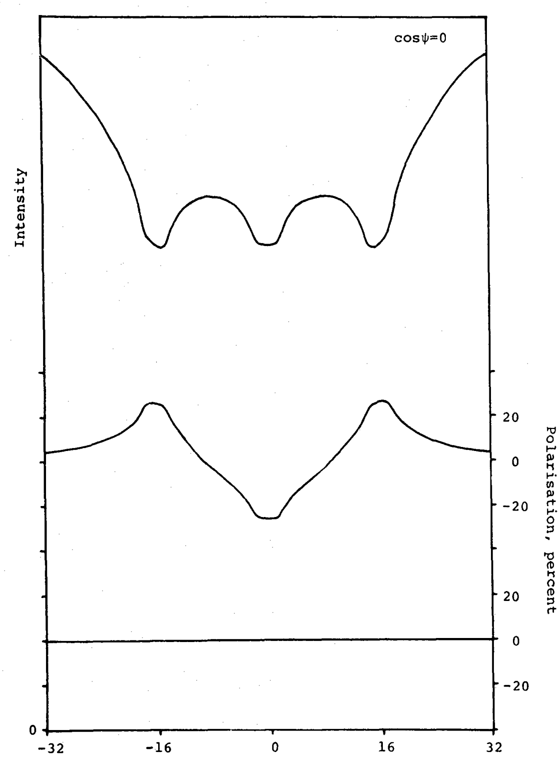

Figure 3. As Fig. 1, but with $\cos\psi = 0$. The solution is qualitatively similar to Unno solution 4, equations (25)–(29).

$$V_5 = -\left(\frac{\rho_R^2 + \rho_R\rho_W \ \eta_Q/\eta_V}{\rho_R^2 + \rho_W^2}\right) B_0\beta\mu\eta_V/D_5, \tag{33}$$

$$D_5 = \eta_I^2 - \frac{(\rho_W\eta_Q + \rho_R\eta_V)^2}{\rho_R^2 + \rho_W^2}. \tag{34}$$

In this approximation, U_5 is very small (of the order η/ρ) and Q_5 and V_5 can assume a wide range of values depending on the signs and magnitudes of η_Q, η_V, ρ_R and ρ_W.

Finally we make the further assumption that $\rho_R^2 \gg \rho_W^2$, which will almost always be the case in the continuum. Then (Unno solution 6)

$$I_6 = B_0(1 + \beta\mu\eta_I/D_6), \tag{35}$$

$$Q_6 = -\left(\frac{\rho_W\eta_V}{\rho_R\eta_Q}\right) B_0\beta\mu\eta_Q/D_6, \tag{36}$$

$$U_6 = \left(\frac{\rho_W\eta_V - \rho_R\eta_Q}{\rho_R^2}\right) B_0\beta\mu\eta_I/D_6, \tag{37}$$

$$V_6 = -B_0\beta\mu\eta_V/D_6, \tag{38}$$

$$D_6 = \eta_I^2 - \eta_V^2. \tag{39}$$

(Note that in 37 we have retained the term $\rho_W\eta_V$ along with $-\rho_R\eta_Q$, since in the continuum both $\rho_R^2 \gg \rho_W^2$ and $\eta_V^2 \gg \eta_Q^2$.) Let us compare Unno solution 6 with Unno solution 1 which applies in the absence of magneto-optical effects. Since in the continuum $\eta_V^2 \gg \eta_Q^2$, $D_6 \simeq D_1$ and so I and V are virtually unaffected by ρ_R and ρ_W when $\rho_R^2 \gg \rho_W^2$. Q_6 is different from Q_1 by a factor $\rho_W\eta_V/(\rho_R\eta_Q)$, while $|U_6| \ll |Q_6|$. Thus in the continuum the expected effect of ρ_R and ρ_W values given by (8) and (9) is to alter the linear polarization by a factor $\rho_W\eta_V/(\rho_R\eta_Q)$ and to leave the intensity and circular polarization essentially unaltered.

In other words, linear polarization in Unno solution 1 is proportional to η_Q while in Unno solution 6 it is proportional to $\rho_W\eta_V/\rho_R$. This is the repolarization effect mentioned earlier. The original emission characteristics represented by η_Q are obliterated by the magneto-optical effects (depolarization), but a new polarization value arises (repolarization) fed by circular polarization emission characteristics represented by η_V via ρ_R and ρ_W into linear polarization. In this limit the ratio of linear to circular polarization is given approximately by ρ_W/ρ_R. Since both ρ_W/ρ_R and η_Q/η_V are of order ω_G/ω in the continuum, the repolarization effect ensures that the absolute value of linear polarization is not drastically changed as a consequence of magneto-optical effects.

3 Results and conclusion

To calculate polarization values in the continuum, we use the zero-field, high-gravity ($\log g$ = 8.0) white dwarf model atmospheres taken from Wickramasinghe (1972) and Wickramasinghe, Cottrell & Bessell (1977) with effective temperature T_e = 20 000 or 6000 K, include polarization of the continuum following Lamb & Sutherland (1974), and assume a magnetic field structure of a centred dipole with polar field strength $B_d = 10^7$ G and 5×10^7 G. Results were calculated with and without the values of the magneto-optical parameters ρ_R and ρ_W from (8) and (9). As expected from Unno solution 6, equations (35)–(39), ρ_R and ρ_W have no effect on circular polarization in the continuum. Results for linear polarization are shown in Table 1.

Table 1. Linear and circular polarization in a magnetic white dwarf with a centred dipole field, with atmospheric structure taken from Wickramasinghe (1972), continuum polarization from Lamb & Sutherland (1974) and no line absorption, for different effective temperatures T_e, polar magnetic fields B_d and viewing angles i. Results for linear polarization are given both for the case with $\rho_R = \rho_W = 0$ (case A) and for the case with ρ_R and ρ_W taken from (8) and (9) (case B). All linear polarization figures should be multiplied by 10^{-5}. Circular polarization results, included here for completeness, are given only for case B, since they are virtually identical in case A; these figures should be multiplied by 10^{-3}.

T_e = 20 000 K	$B_d = 10^7$ G	λ = 3500 Å	4500 Å	5500 Å	6500 Å	7500 Å
$i = 0°$	linear A	0	0	0	0	0
	linear B	0	0	0	0	0
	circular	−4.9	−5.3	−5.3	−5.2	−5.1
$i = 45°$	linear A	2.5	4.2	3.7	3.4	2.9
	linear B	−0.84	−1.05	−1.25	−1.45	−1.64
	circular	−3.5	−3.7	−3.7	−3.7	−3.6
$i = 90°$	linear A	5.1	8.4	7.4	6.8	5.8
	linear B	−1.7	−2.3	−2.8	−3.3	−3.7
	circular	0	0	0	0	0
T_e = 20 000 K	$B_d = 5 \times 10^7$ G					
$i = 45°$	linear A	–	42	44	47	–
	linear B	–	−21	−25	−29	–
	circular	–	−18	−18	−17	–
T_e = 6000 K,	$B_d = 10^7$ G					
$i = 0°$	linear A	0	0	0	0	0
	linear B	0	0	0	0	0
	circular	−3.9	−4.0	−3.7	−3.3	−2.7
$i = 45°$	linear A	1.62	0.41	−0.32	−0.75	−1.15
	linear B	1.04	−0.27	−1.05	−1.49	−1.72
	circular	−2.7	−2.9	−2.6	−2.3	−1.9
$i = 90°$	linear A	3.3	0.82	−0.66	−1.53	−2.3
	linear B	3.2	0.68	−0.84	−1.75	−2.5
	circular	0	0	0	0	0

When the viewing angle i is significantly different from zero, linear polarization is produced when $\rho_R = \rho_W = 0$. When values for these parameters due to free electrons are included – equations (8) and (9) – linear polarization due to characteristics represented by η_Q is obliterated (depolarization) and linear polarization deriving from circular polarization via the magneto-optical parameters is created (repolarization). For T_e = 20 000 K the repolarization happens to be smaller in magnitude and opposite in sign to the values prior to depolarization, while for T_e = 6000 K the different opacities and lower electron densities lead to a more complex effect apparently combining partial depolarization and partial repolarization. (At all temperatures depolarization is not complete when an average over the surface of the star is taken, since for angles near $i = 90°$ linear depolarization does not fully occur, as shown in Fig. 3.) At the higher polar field strength of 5×10^7 G the opacity shifts of Lamb & Sutherland (1974) are less likely to be accurate; furthermore, the shifts are so large that λ = 3500 and 7500 Å require opacity values outside those provided in Wickramasinghe (1972), and hence are not shown. However, it is clear from these calculations that even at these high fields the linear polarization remains small compared to the circular polarization as is expected from Unno solution 6 discussed earlier. The high values of linear polarization – similar in magnitude to circular polarization – observed in some magnetic white dwarfs must imply even higher fields ($\gtrsim 10^8$ G). A proper treatment of

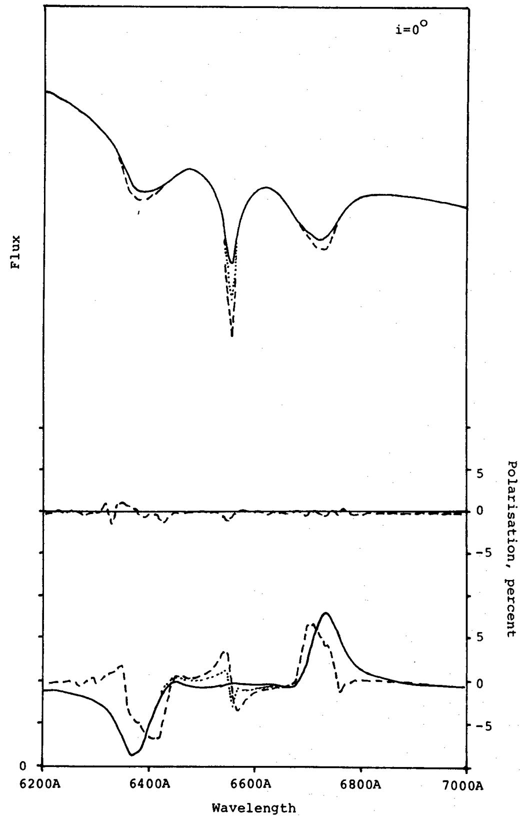

Figure 4. Flux (top), linear polarization (middle) and circular polarization (bottom) for a realistic model magnetic white dwarf near the Hα absorption line. The solid line is the solution with $\rho_R = \rho_W = 0$, the dotted line is the solution with ρ_R and ρ_W from the Hα line only (shown only when distinguishable from the solid line), and the dashed line is the solution with ρ_R and ρ_W contributions from the lines and from free electrons in the continuum (shown only when distinguishable from the solid line). The white dwarf model atmosphere is the zero-field, high-gravity ($\log g = 8.0$), $T_e = 20\,000$ K model taken from Wickramasinghe (1972), the continuum is polarized following Lamb & Sutherland (1974), and the pole strength of the assumed centred dipole field is taken to be $B_d = 10^7$ G. The star is viewed at an angle $i = 0°$ with respect to the dipole axis (pole-on).

these high field cases requires a different formulation of the problem which includes cyclotron opacity.

The effects of free-electron magneto-optical effects on line spectra are illustrated in Figs 4, 5 and 6. The calculations were carried out similarly to the continuum calculations, but with the inclusion of the full set of components for the hydrogen absorption line Hα using shifts from Kemic (1974). The Voigt line profiles and values for ρ_R and ρ_W associated with the lines are as in Paper I. Results are given for the case with $\rho_R = \rho_W = 0$, for the case with ρ_R and ρ_W from lines only, and for the case with ρ_R and ρ_W from lines and free electrons.

The free-electron magneto-optical effects clearly have a greater effect on the spectrum than the magneto-optical effects associated with the lines. The lines are deepened, especially the central component of the Zeeman triplet; the magnitude of circular polarization in the outer components is reduced and the position of greatest polarization is shifted inwards, while the bulge at the central component is accentuated; and linear polarization is greatly reduced, though not obliterated. Most of these results are readily understandable as a superposition of solutions pictured in Figs 1 to 3. Note that the average magnitude of polarization in Figs 4 to 6 is much smaller than in Figs 1 to 3; this results from averaging the solution over the surface of the star.

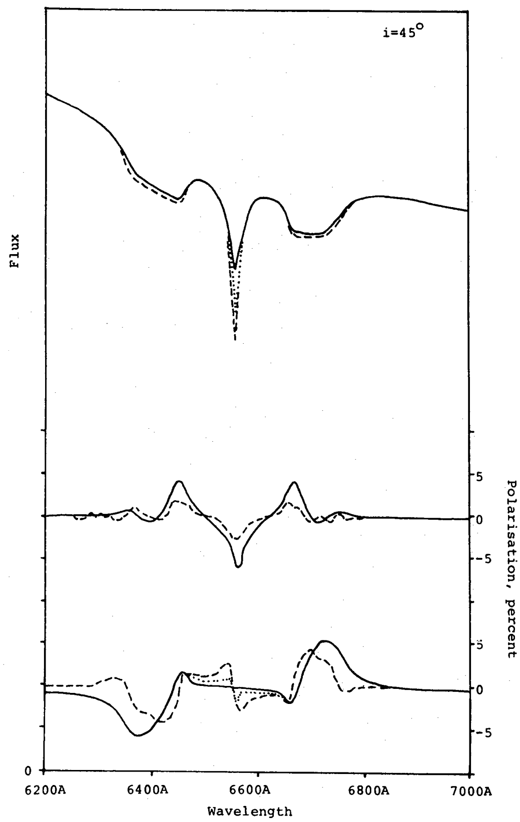

Figure 5. As Fig. 4, with viewing angle $i = 45°$.

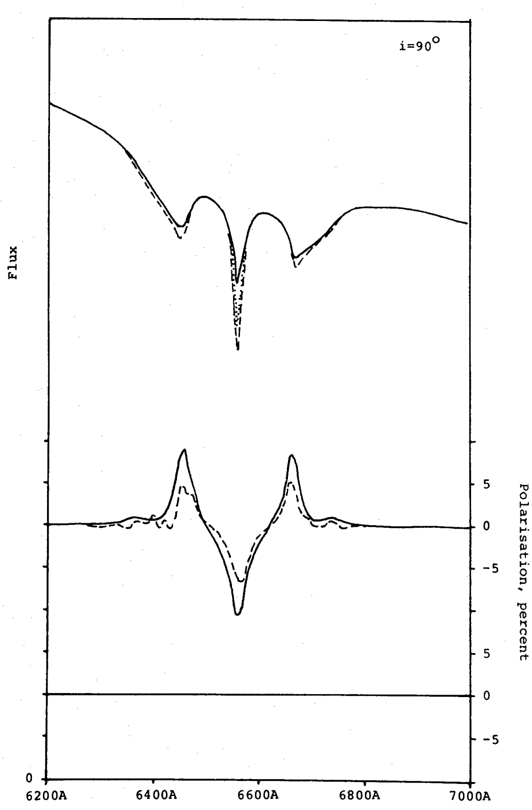

Figure 6. As Fig. 4, with viewing angle $i = 90°$ (equator-on).

When lines are present, the values for linear polarization in the continuum given in Table 1 of course no longer apply except in regions away from the line opacities and line-based magneto-optical effects. For $B_d = 10^7$ G only the regions of $5500 < \lambda < 6000$ Å and $7000 \text{Å} < \lambda$ are unaffected by hydrogen lines. Values of circular polarization in the continuum are not affected by magneto-optical effects, which therefore alter circular polarization mainly in the regions of the principal line components as shown in Figs 4 and 5.

There is one major obstacle encountered in calculating results such as in Fig. 4 to 6: numerical inaccuracy, especially in determining linear polarization. At each different latitude on the surface of the star, the magnetic field strength is different and hence the line shifts determined by Kemic (1974) are different. If the number of latitudes used in the numerical integration over the surface of the star is too small, inaccuracy can result because the magnetic field strengths and shifts involved may be unrepresentative. This problem arises in all calculations of the type we have undertaken (Martin & Wickramasinghe 1978; Wickramasinghe & Martin 1979; Paper I), but is much more serious when the large values of ρ_R and ρ_W from (8) and (9) are included, because the variability of parameter values affecting the solution to (1)–(4) from one latitude to another is much greater. (Variations in parameter values from one longitude to another do not create such serious difficulties.) Furthermore, the reduced value of linear polarization, which results from depolarization and repolarization, is much more sensitive to numerical inaccuracies. To obtain reasonably stable results, we use 100 latitudes (pole to pole) in our calculations for Figs 4 to 6. The required computer time is sizeable, and precluded similar calculations for other lines and more wavelengths. Even so, the values for linear polarization in the lines obtained are probably only meaningful when they are greater than a few tenths of a per cent.

If different line profiles were used, the resulting linear polarization could be rather different due to the sensitivity of the solutions to opacity parameters when ρ_R and ρ_w are large or rapidly varying. Hence our results in the lines, for linear polarization in particular, should be seen mainly as illustrating the general features of the impact of magneto-optical effects from free electrons on the line spectra of magnetic white dwarfs, rather than as providing detailed quantitative predictions.

The results obtained here should sound a cautionary note concerning interpretation of spectra of magnetic white dwarfs. In Paper I we described the impact of magneto-optical effects arising from absorption lines. Yet those effects are overshadowed by magneto-optical effects due to free electrons, as we have shown in this paper. It would be presumptuous to claim that all the effects, magneto-optical or otherwise, which are necessary to model magnetic white dwarfs have been properly assessed, or even yet discovered.

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