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Galactic

Cyclotron Absorption in GD229?

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Introduction

The existence of a group of magnetic white dwarfs with mean surface fields of between $\sim 5 \times 10^6$ G and $\sim 2 \times 10^7$ G is now firmly established through the discovery of Zeeman structure in hydrogen and helium lines (Angel *et al.* 1974, Liebert *et al.* 1975, Wickramasinghe *et al.* 1977, Martin & Wickramasinghe 1978, Liebert *et al.* 1977). There is considerable evidence from polarimetric studies for even higher fields in some white dwarfs (Angel 1977). Most of these stars are cool and show nearly continuous spectra or weak unidentified absorption features. The possibility of cyclotron absorption in white dwarfs was first discussed in connection with the infrared polarisation spectrum of one of these stars, Grw + 70°8247 (Kemp 1970). More recently the polarised white dwarf GD229 was found to have a rich optical spectrum with a strong feature at λ4185, for which cyclotron absorption has been mentioned as a possible origin (Angel 1977, Greenstein & Boksenberg 1977), though without supporting computations. In this letter we use models to investigate this possibility further.

GD229 was listed as a white dwarf suspect in the Lowell proper motion survey (Giclas *et al.* 1965) and discovered to exhibit wavelength dependent linear and circular polarisation in 1974 (Swedlund *et al.* 1974, Landstreet & Angel 1974). Its peculiar spectrum was described by Greenstein *et al.* (1974) and has been studied subsequently by a number of investigators (Greenstein & Boksenberg 1977, Liebert 1976). Image Dissector Scanner observations published by Liebert (1976) are reproduced in Fig. 1. Weaker features have also been reported blueward of λ4280 and redward of λ6000 (Greenstein & Boksenberg 1977).

Calculations

The cyclotron absorption cross section per electron for transitions between adjacent Landau levels has been calculated quantum mechanically by Lamb & Sutherland (1974) to be

$$\sigma_+(\omega,\theta) \simeq 4\pi^{3/2} \frac{e^2}{m_e c} \frac{1}{\omega} \left(\frac{m_e c^2}{2kT\cos^2\theta}\right)^{1/2} \times$$

$$\times \left(1 - \epsilon^{-\hbar\omega/kT-1}\right) \exp\left(-\frac{m_e c^2}{2kT\cos^2\theta} \frac{(\omega-\omega_c)^2}{\omega^2}\right),$$

where ω is the angular frequency, $\omega_c = eB/m_e c$ is the cyclotron frequency, and θ is the angle between the field direction and the angle of propagation. This formula does not allow for collisional broadening of the levels, which could be important. Unfortunately a quantum mechanical treatment of the latter problem is not available.

The cyclotron absorption, which affects only right-handed polarised light, differs in form greatly from line absorption in being both extremely strong (at least where the number density of electrons is high) and extremely narrow (due to the final exponential factor in σ_+). The net effect is reduction in intensity from the star at any given wavelength by saturation or near saturation of only a small portion of the disc. To reproduce anything like the observed depth and width of the λ4185 feature therefore requires a high degree of uniformity in field strength.

Our results are based on a two component model of the field, consisting of a uniform field superimposed on a centred dipole field distribution whose polar strength equals the uniform field strength. These two components are weighted by fractions R and $1 - R$ respectively. R therefore measures the degree of uniformity of the field distribution, and is a quantity on which we can place limits by comparison with observations. (Since we find that the uniform component is dominant, the orientation of the dipole is not critical and is assumed for

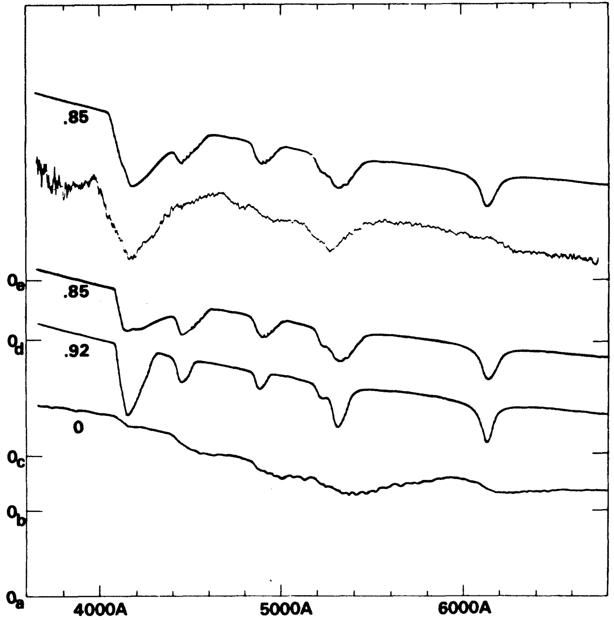

Fig. 1. Observed and theoretical intensity spectra as a function of wavelength in angstroms for GD229. The zeros for the 5 curves are given on the left vertical axis. The observed spectrum (d) is the average of 5 sets of observations by Liebert. The other theoretical curves are labelled by the degree of field uniformity R assumed. The top theoretical curve (e) also assumes a doubling of the half width for cyclotron absorption, to suggest the possible effect of including collisional broadening. The Hα components causing the observed dips in (b), (c), and (e) are (from right to left), 2s0 – 3p-1 (σ_+), 2p + 1 – 3s0 (σ_+), 2s0 – 3p0 (π, overlapping previous σ_+), 2p0 – 3d0 (π), and 2p + 1 – 3d + 1 and 2p – 1 – 3d – 1 (both identically located π, overlapping cyclotron absorption). These computations are based on linear extrapolation in Kemic's (1974) tables.

simplicity to coincide with the direction of the uniform component.)

It should be emphasised that our two-component model of the magnetic field is used primarily for conceptual and computational convenience. An actual near-uniform field could arise for a number of reasons. One possibility is an off-set dipole: if the dipole is off-set by 0.3 of the radius or more, the field on the side of weakest field is remarkably uniform. Another possibility is that the flux from the polar region of the star is abnormally high due to enhanced electron conduction along field lines in the envelope and interior.

The method we employ to compute theoretical spectra for magnetic white dwarfs has been discussed in detail elsewhere (Martin & Wickramasinghe 1978). In essence we divide the surface of the star into a number of segments, compute the emergent polarisation spectrum in each segment, and suitably average over the visible stellar disc. The radiative transfer equations are formulated in terms of Stokes parameters (Unno 1956) and solved numerically assuming that the temperature and pressure structure of the atmosphere is the same as that of a zero field white dwarf of the same effective temperature.

In the present case we have used zero field values for the continuous opacity since an appropriate theory valid for $B \gtrsim 10^8$ is not available. Fortunately, sizable variations in the continuous opacity do not greatly affect the results for the depth and the width of the cyclotron feature due to the peculiar exponential wing of the cyclotron absorption cross section (1). However, because polarisation is more sensitive to the continuous opacity, our treatment of the continuous opacity does not permit comparison with observed polarisation data, though we have allowed explicitly for polarisation of the cyclotron absorption feature.

We discuss a series of models computed assuming $T_e = 12000$ K and $\log g = 8.0$ (Wickramasinghe 1972), which yields a continuum slope consistent with Liebert's (1976) IDS data. The effects of magnetic broadening are illustrated in Fig. 1 where we present theoretical spectra computed for various values of R. In each model the polar field strength is 2.6×10^8 G and the angle between the line of sight and the magnetic axis is set equal to zero.

Results and conclusions

We find that for a pure dipole field ($R = 0$) the cyclotron feature is broadened several thousand angstroms and rendered undetectable. The broad weak absorption features are caused by the Hα components which are discussed below. As R is increased, the maximum depth d increases while the full half width $w_{0 \cdot 5}$ decreases; at $R = 0.92$ a narrow ($w_{0 \cdot 5} \sim 140$A) deep ($d \sim 43\%$) absorption results. The steep blueward edge is a characteristic of cyclotron absorption and is caused by the second exponential factor in σ_+ (eqn (1)), as discussed earlier. Similar results hold when the line of sight is not parallel to the magnetic axis, but the feature is intrinsically weaker.

The computed profiles for $R \sim 0.85$ are similar in many respects to the observed λ4185 feature. However several details need to be clarified. In particular the models do not predict a cyclotron feature which is as wide ($w_{0 \cdot 5} \sim 250$A) and deep ($d \sim 46\%$) as observed. This problem could be resolved if the cyclotron cross section σ_+ were somewhat wider than we have assumed, which will be the case if collisional broadening is important. Increasing the half width by a factor 2 has a marked effect on the cyclotron feature (as also shown in Fig. 1), and yields a profile in reasonable agreement with observations considering the number of uncertainties in the calculation. However, the precise manner in which the shape of σ_+ is affected by collisional broadening is unknown and must await quantum mechanical calculations. (Another more remote possibility is that the star is considerably hotter (~ 35000 K) than our unshifted continuum calculations and the IDS data indicate. This would also have the desired effect of increasing the half width of $\sigma_+(\omega,\theta)$.)

We have also computed the Hα spectrum for our models using Kemic's (1974) computations. For field strengths relevant to GD229 ($B > 10^8$ G) the wavelengths have been extrapolated linearly while the intensities have been kept equal to their value at 10^8 G. This procedure, though reasonable, cannot entirely be justified, particularly since the assumptions involved in Kemic's (1974) computations are based on a perturbation approximation which is expected to break down for Hα at $B \gtrsim 10^8$ G. The variational calculations of Smith *et al.* (1972) and Praddaude (1972) are perhaps more relevant at $B > 10^8$ G and the few available results suggest a more complicated behaviour with several components of Hα being more stationary than suggested by extrapolation in Kemic's tables (Angel 1978). Bearing in mind these uncertainties, the general agreement between theory and observations is encouraging and suggests that the remaining features in the spectrum of G D229 may be due to bound-bound transitions of hydrogen. We note in particular that with the adopted field distribution, the Hα components remain detectable at approximately the observed central depths. However, any detailed agreement must be considered fortuitous until accurate computations of strengths and shifts of Zeeman components of Balmer lines for fields of up to $\sim 3 \times 10^8$ G become available.

Our interpretation of the λ4185 feature in the spectrum of GD229, it should be noted, depends on two important assumptions. One is the high uniformity of the magnetic field, whether this be due to an off-centred dipole, to high polar flux, or to another reason. Our two component model is meant to characterise in a straightforward manner this assumed high uniformity of the field. Second is the broadening of the cyclotron cross section. Nevertheless we feel that the sharp blueward edge of the λ4185 feature and its strength favours a cyclotron interpretation.

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New Observations of the Beat Cepheid U Trianguli Australis

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The double-mode or beat Cepheids continue to pose a number of interesting questions in our search for an understanding of the pulsation properties of stars near the short period end of the instability strip. One such is how simultaneous pulsations in the fundamental and first overtone radial modes can occur for an appreciable fraction of Cepheids in the period range $1\text{d} < P_0 < 5\text{d}$.

Two possible explanations have been proposed (Stellingwerf 1975b). Perhaps the simpler of the two is that these stars might be "mode switching" as their evolution carries them across a boundary between regions of fundamental and first overtone instability. Stellingwerf has used the rate of evolution of Cepheid models calculated by Iben (1967), and his own non-linear switching rate calculations to predict an e-folding time of ~ 80 yr for the growth of the first overtone at the expense of the fundamental in a blueward crossing of this boundary. A redward crossing (first overtone to fundamental) is expected to be even more rapid. The second explanation is that the beat Cepheids might occupy a region of the Hertzsprung-Russell diagram where continuing mixed-mode behaviour is possible. Stellingwerf (1975a) has found evidence for the existence of such a domain near the red edge of the instability strip in his study of Population II, RR Lyrae models, but there is a paucity of similar evidence for stellar models with properties matching those of the beat Cepheids themselves.

At first sight, the existence or otherwise of an appreciable change, say 20%, in the relative energies of the two modes over a timescale of ~ 20yr might appear a criterion for distinguishing between these two hypotheses. There are many physical systems, however, in which coupling between the modes of oscillation leads to a continual transfer of energy back and forth from mode to mode over timescales determined by the mode periods, the degree of intermode coupling, etc. (e.g., Sommerfield 1952). Faulkner (1977a,c) has demonstrated a considerable amount of mode interaction in the beat Cepheids U TrA and TU Cas, so a change of relative pulsation energies might be expected even on the continuing multi-mode explanation for these stars. The detection of a change in the relative mode energies, while not distinguishing between the two proposed explanations, places important constraints upon them, and we report such a detection for U TrA in this paper.

U Trianguli Australis was extensively observed in the years 1953-59 (Oosterhoff 1957; Jansen 1962), and these measures were recently Fourier analyzed by Faulkner (1977a). He found that Fourier terms to at least fourth order were conspicuous in the analysis, and that the mode interaction terms were quite appreciable compared with those associated with the two periodicities themselves. In 1977 we obtained a new, extensive series of observations of U TrA, using the Visvanathan fast digital photoelectric photometer (Visvanathan 1972) on the 60-cm reflector at Siding Spring. Four hundred and eighty one UBVRI observations were made over a three month period (Fig. 1). These measures will be described in full elsewhere; the

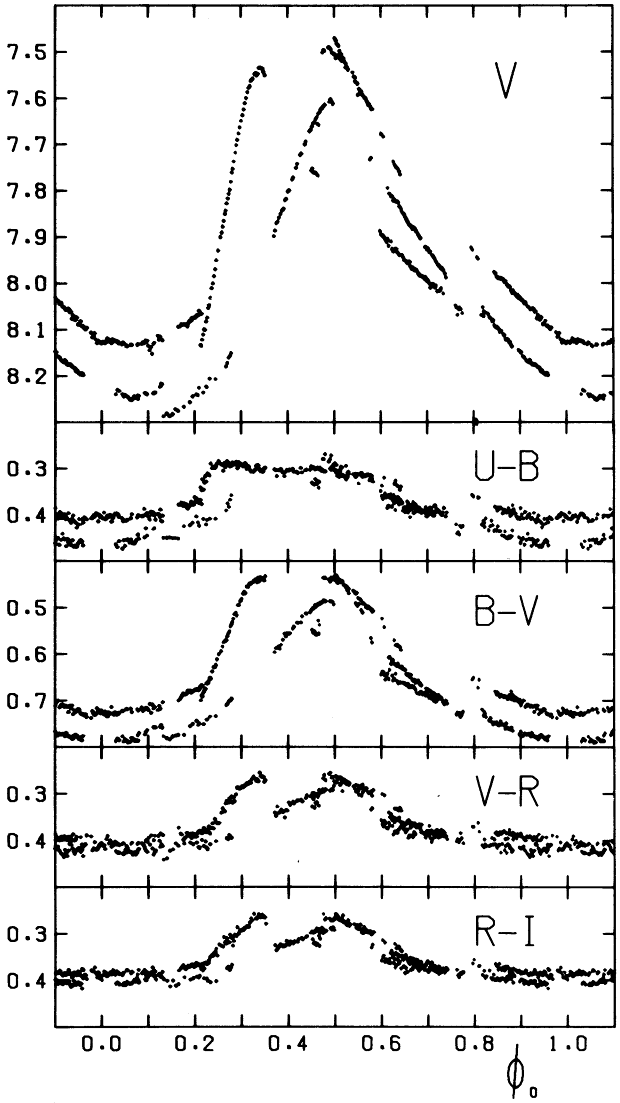

Figure 1. UBVRI observations of U TrA made in 1977, plotted against the phase of the primary pulsation.