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Cyclotron absorption in magnetic white dwarfs

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Summary. The possible effect of cyclotron absorption in highly magnetic white dwarfs is investigated using model atmospheres. For a dipole field geometry, Doppler broadened cyclotron absorption is unlikely to produce significant features at all, while collisionally broadened cyclotron absorption can produce a broad shallow absorption feature which may become more significant in the case of birefringence and scattering or of a very steep temperature gradient near the stellar surface. The broad absorption feature and circular polarization of G240 – 72 may be explained through these latter mechanisms.

1 Introduction

In a recent review, Angel (1978) discussed 16 white dwarf stars with high magnetic fields ($> 10^6$ G). Some of these have identifiable Zeeman structure in absorption lines, from which the magnetic field strength may be deduced. Other white dwarfs give evidence of a magnetic field only through polarization: the spectrum is either featureless or obscured by a more highly luminous binary partner. Still other white dwarfs show both polarization indicating a high magnetic field, and absorption features which remain unidentified: GD229, G240 – 72, LP790 – 29 and Grw + 70° 8247 are examples.

It has been suggested (Kemp 1970; Lamb & Sutherland 1974; Angel 1977, 1978; Liebert, Angel & Stockman 1978; Wickramasinghe & Martin 1978) that cyclotron absorption may play a role in some of these latter sorts of stars. In this paper we investigate the characteristics of cyclotron absorption in terms of its potential role in magnetic white dwarfs. We first describe the mechanisms involved, and then present results of calculations using a model atmosphere.

2 Characteristics of cyclotron absorption

The cross-section for absorption due to free–free transitions by electrons between adjacent Landau levels in a magnetic field has been calculated quantum mechanically and approximately by Lamb & Sutherland (1971) and presented by Lamb & Sutherland (1974):

$$\sigma_+(\omega, \psi) = 4\pi^{3/2} \frac{e^2}{mc} \frac{1}{\omega} \left(\frac{mc^2}{2kT \cos^2 \psi} \right)^{1/2} \times \frac{1}{1 - \exp(-\hbar\omega/kT)} \exp\left(-\frac{mc^2}{2kT \cos^2 \psi} \frac{(\omega - \Omega_c)^2}{\omega^2} \right), \quad (1)$$

in which light of frequency ω propagates at an angle ψ with respect to the magnetic field, the strength of which enters equation (1) through the cyclotron frequency $\Omega_c = eB/mc$. Since σ_+ is negligible except near $\omega = \Omega_c$, Lamb & Sutherland (1971, 1974) substituted Ω_c for ω in their expression for σ_+ except not of course in the factor $(\omega - \Omega_c)^2$. The cross-section applies only to right hand circularly polarized light.

The strength of this cyclotron absorption is very great, but its width is very narrow due to the final exponential term. In terms of absorption features in a magnetic white dwarf, the effect of cyclotron absorption may be looked at profitably in the following way. First, the cyclotron absorption is almost completely effective within roughly 3 half widths of the resonance, but is almost negligible outside roughly 4 half widths. (Assume the cyclotron opacity at $\omega = \Omega_c$ is r times the continuum opacity; at n times the half width at $1/e$ of the maximum, the ratio of opacities will be $r \exp(-n^2)$. For $r = 10^6$ (a typical value in a white dwarf atmosphere) and $n = 3$, cyclotron absorption is dominant; by $n = 4$ cyclotron absorption has become negligible. Large changes in r have relatively little effect on these values of n.)

Second, a large value of σ_+ results in only 50 per cent saturation of the spectrum, because only right hand circularly polarized light is absorbed. Therefore, it is impossible for cyclotron absorption alone to lead to lines more than 50 per cent deep, and in most stars the theoretical maximum depth will be much less than this. (If birefringence or scattering are important, this conclusion does not hold and up to 100 per cent saturation can be obtained locally.)

Third, the effect of cyclotron absorption in a magnetic white dwarf will depend on the fraction of the visible disc over which the absorption is operative. Noting that $\omega \approx \Omega_c$ near the cyclotron resonance, the half width in terms of magnetic field is

$$\frac{\Delta\Omega_c}{\Omega_c} = \left(\frac{2kT}{mc^2} \right)^{1/2} |\cos \psi|. \quad (2)$$

For a dipole field, the latitudinal angular half width of the absorption is, for latitude θ,

$$\Delta\theta = \frac{1/3 + \cos^2 \theta}{\sin\theta \cos\theta} D, \quad (3)$$

where $D = (2kT/mc^2)^{1/2} \cos \psi$. This approximation obviously breaks down near the pole or equator: at $\theta = 0$, $\Delta\theta = (8D/3)^{1/2}$, and at $\theta = \pi/2$, $\Delta\theta = (2D/3)^{1/2}$. Taking into account also the surface area associated with different latitude bands, it is found that for $T = 10\,000$ K and wavelengths such that the cyclotron resonance is centred at $\theta = 0$, $\pi/4$ and $\pi/2$, the fraction of the surface area of the star within 4 half widths of the resonance is 0.04 $(\cos \psi)^{1/2}$, $0.08 \cos \psi$ and $0.29 (\cos \psi)^{1/2}$ respectively. The effect of the absorption of course also depends on the viewing angle (which, with the latitude and longitude, determines ψ), on the pressure and temperature structure of the atmosphere which determine the τ dependence of the continuum and cyclotron opacities, and on the precise solution to the radiative transfer equations in the regime where cyclotron absorption is neither dominant nor negligible. Because of the complexity of these factors, we do not present further intermediate results. However, the above simple considerations tell us that cyclotron absorption potentially has effects over a range of wavelengths which vary by a factor slightly greater than 2 (corresponding to the factor of 2 variation in magnetic field strength in a dipole field), and that significant effects are likely to be present only near $\omega = \Omega_c/2$ (corresponding to the magnetic field at the equator), assuming that the viewing angle is such that $\cos\psi$ is not too small and that the equatorial region is not solely on the limb.

Another way in which cyclotron opacity can arise is through collisional broadening. A classically derived expression (Bekefi 1966)* for the absorption cross section is

$$\sigma_+^c(\omega, \psi) = \frac{e^2}{mc}(1+\cos^2\psi)\frac{kT}{2\pi\hbar\omega}[\exp(\hbar\omega/kT)-1]\frac{\nu}{(\omega-\Omega_c)^2+\nu^2}, \quad (4)$$

where ν is the collision rate.

The strength of the collisional cyclotron absorption is, like that of equation (1), very great; in this case the width depends on ν. For small ν the width is very narrow and the absorption effect is limited by the considerations noted for the case of Doppler broadened absorption. For large ν, on the other hand, σ_+^c becomes so broad that once again no significant absorption *feature* appears: a large fraction of the spectrum is depressed (though not by a large amount due to absorption only of right hand circularly polarized light). The effect for large ν may be looked at in the following way. Assume the value of ν is such that, at a given wavelength, there is significant absorption over one half the visible disc of a magnetic white dwarf with a dipole field. Then at a somewhat different wavelength, there will still be absorption over quite close to one half the visible disc, and hence no dramatic change in the depth of the feature. Obviously the same considerations apply if significant absorption occurs over less or more than one half the visible disc. Because the field varies by a factor of 2 across the surface of the star, absorption will occur roughly over a range of wavelengths varying by a factor 2; if ν is sufficiently large, the only possible noticeable features will be near $\omega = \Omega_c$ and $\omega = \Omega_c/2$, where the absorption feature begins and ends. Between these wavelengths the spectrum will be depressed but smooth, and even the beginning and end of the depression may be washed out, depending on the viewing angle and other factors.

If both Doppler and collisional broadening are operative, the cross section must be evaluated in terms of a Voigt function, in which near the line centre the Doppler shape (1) dominates, while in the wings the Lorentz shape (4) dominates.

3 Illustrative model results

To give an idea of the impact of the various parameters involved on cyclotron absorption, we present results here of calculations done using a model white dwarf atmosphere. The pressure and temperature structure for a 6000 K, $\log g = 8.0$ DA white dwarf with a metal abundance of 10^{-3} the solar value are taken from Wickramasinghe, Cottrell & Bessell (1977). This atmosphere is adopted mainly because it is available; other model atmospheres would show similar effects. The field is taken to be a centred dipole of strength 3.8×10^8 G, viewed at an angle $i = 75°$ with respect to the magnetic axis. The radiative transfer equations for the Stokes parameters *I, Q* and *V* are solved using the method of Martin & Wickramasinghe (1979). To simplify the calculation of the cyclotron absorption cross section, the larger of

* Bekefi (1966) also derives a classical expression for Doppler broadened cyclotron absorption, which can differ (depending on the wavelength and temperature) by up to a factor of 10 or 100 from the quantum mechanical expression of Lamb & Sutherland (1974). However, the strength and shape of cyclotron absorption is such that qualitative effects would be unaltered by a sizeable change in absorption strength, and even quantitative results would be virtually unchanged (see Table 1).

the Doppler and collisional cross sections (1) and (4) was used at any given wavelength and latitude. In fact, tests show that ignoring the Doppler cross section altogether leads to only negligible changes in results for the range of ν values considered here (except $\nu = 0$). Therefore the approach used should give a good approximation to a Voigt profile.

The integration over the surface of the star is performed using a number of different latitudes and longitudes as in Martin & Wickramasinghe (1978), except that a higher density of integration points is used near the latitude at which the cyclotron resonance occurs. No account is taken of the polarization in the continuum, both to simplify interpretation of the results and because the available treatment (Lamb & Sutherland 1974) breaks down for such high magnetic fields. (We note that in wavelength regions where collisionally broadened cyclotron absorption is the dominant opacity source over the entire surface of the star, changes in the continuum opacities due to polarization will have only a very small effect on flux and polarization values.) For other details of the model computations, see Martin & Wickramasinghe (1978).

In Table 1 the fractional depression of the spectrum for a number of cases is presented. This mode of presentation is adopted because the effects are usually rather unimpressive visually. Selected theoretical spectra are plotted in Fig. 1.

With Doppler broadening only ($\nu = 0$), there is only a small localized depression in the spectrum, caused by the latitude region near the equator. The collision rate which maximizes the visual effect of an absorption feature is near $\nu = 5 \times 10^{11}$/s. Even in this case, the feature is very broad and shallow. Increasing the collisional rate beyond this ($\nu = 5 \times 10^{12}$/s) begins to flatten the entire spectrum, while reducing it ($\nu = 5 \times 10^{10}$/s) also reduces the feature to insignificance. Because increasing the collision rate beyond $\nu = 5 \times 10^{11}$/s increases the cyclotron opacity outside the core almost proportionately to ν, this is almost equivalent to increasing the total cyclotron strength by the same factor. We can draw the conclusion that changes in the total strength of cyclotron absorption will introduce no new qualitative effects into the resulting spectrum. For different viewing angles ($i = 0°, 45°, 90°$), the same portions of the disc are saturated, but they are given different weights in summing up the net absorption; no dramatic absorption effects are expected by altering the viewing angle.

Table 1. Fractional reduction in flux due to cyclotron absorption for a number of cases. Unless otherwise indicated, $B_d = 3.8 \times 10^8$G, $i = 75°$, $\nu = 5 \times 10^{11}$/s, $T_e = 6000$ K, $T_s = 4881$ K, $f_U = 0$.

Case	Wavelength 4000Å	4500Å	5000Å	5500Å	6000Å
$B_d=0$	0	0	0	0	0
$\nu=0$	.004	.003	.003	.003	0
$\nu=5\times10^{11}s^{-1}$	.09	.07	.07	.08	.01
$\nu=5\times10^{12}s^{-1}$	.22	.19	.17	.14	.06
$\nu=5\times10^{10}s^{-1}$	.03	.02	.02	.03	.001
Cyclotron strength increased by factor of 10	.22	.19	.17	.14	.06
$i=0^o$	.08	.04	.02	.01	.001
$i=45^o$	.08	.05	.05	.05	.01
$i=90^o$	.09	.08	.07	.09	.01
$B_d=2\times10^8G$, $f_U=0.9$	.01	.01	.04	.27	.03
$T_s=4000K$	.14	.11	.10	.12	.02
$T_s=3000K$	.35	.28	.23	.19	.05

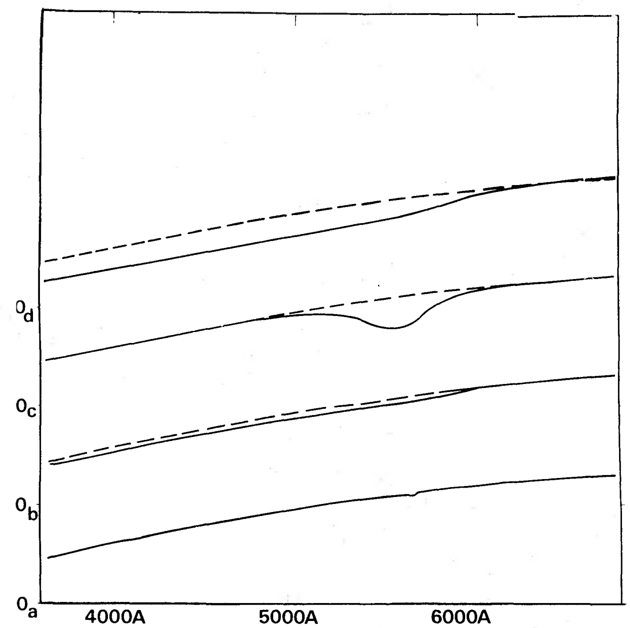

Figure 1. Flux for a model 6000 K magnetic white dwarf in the presence of cyclotron absorption. The zero points of the four curves are labelled a to d from the bottom up. Parameters for the solid curves are: (a) $B_d = 3.8 \times 10^8$ G, $i = 75°$, $\nu = 0$ (Doppler broadening only), $T_s = 4881$ K (standard temperature structure), $f_U = 0$ (dipole field); (b) as (a) except $\nu = 5 \times 10^{11}$/s; (c) $B_d = 2 \times 10^8$ G, $i = 75°$, $\nu = 5 \times 10^{11}$/s, $T_s = 4881$ K, $f_U = 0.9$; (d) $B_d = 3.8 \times 10^8$ G, $i = 75°$, $\nu = 5 \times 10^{11}$/s, $T_s = 3000$ K, $f_U = 0$. The dashed lines represent the flux with no cyclotron absorption ($B_d = 0$).

There seem to be several ways through which cyclotron absorption can give rise to more distinctive absorption features. First, if birefringence or scattering play a major role, the cyclotron absorption can result in up to 100 per cent saturation of the spectrum locally rather than a maximum of 50 per cent. Second, a more uniform magnetic field can give rise to a deeper feature due to the increased fraction of the visible disc over which significant absorption occurs. This effect occurs however at the expense of the breadth of the feature. This possibility has been analysed in some detail in Wickramasinghe & Martin (1978). In Table 1 and Fig. 1 this effect is illustrated by making the field 90 per cent uniform in the direction of the dipole ($f_U = 0.9$) and 10 per cent dipole. Because of the higher average field strength, the dipole (and uniform) field strength is reduced to 2×10^8 G to move the absorption feature to a place in the spectrum similar to that of the features for the pure dipole cases.

A further way in which a more significant absorption feature may arise is through an increased temperature gradient at low optical depths ($\tau \lesssim 0.01$). This might occur through increased blanketing as the collisionally broadened cyclotron absorbs across a wide range of the spectrum, or as a result of the magnetic field having a direct influence on the atmospheric structure. To test the effect such a change in temperature structure might have, we have changed the gradient only in the region $0 < \tau < 0.001$ by reducing the surface temperature below its model value of 4881 K. The result, as seen in Table 1 and Fig. 1, is a deepened broad feature extending blueward of roughly 5500 Å.

The possibilities of birefringence and scattering and of a steeper temperature gradient make collisionally broadened cyclotron absorption a candidate for the explanation of the broad shallow absorption feature in G240 – 72 (Greenstein 1974; Liebert 1976), whose spectrum is presented in Fig. 2. We present also in Fig. 2 a theoretical model with $\nu = 5 \times 10^{11}$/s and a surface temperature of 3000 K. (It is also possible that birefringence or

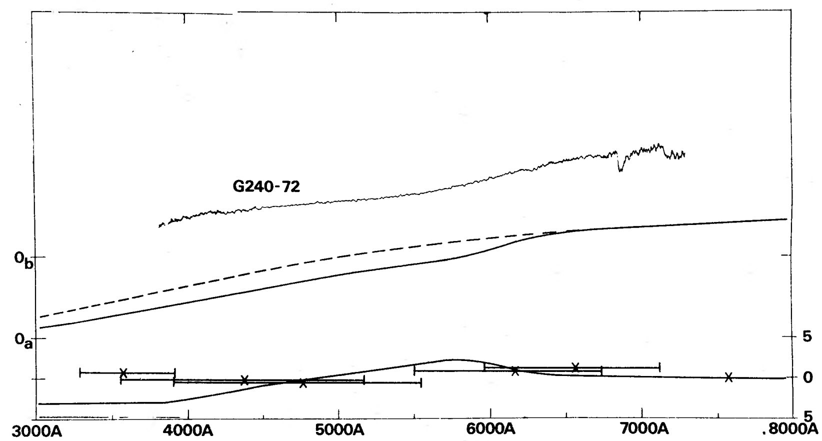

Figure 2. Observed and theoretical fluxes and circular polarization for G240 – 72. The top curve is flux measurements from Liebert (1976). The curve below it represents flux for a 6000 K model atmosphere with $B_d = 3.6 \times 10^8$ G, $i = 85°$, $\nu = 5 \times 10^{11}$/s, $T_s = 3000$ K (very steep surface temperature gradient) and $f_U = 0$ (dipole field), normalized at 7100 Å to the flux observed. The dashed curve indicates a model atmosphere with no cyclotron absorption ($B_d = 0$). The zeros for these two curves are noted 0_b and 0_a respectively. The bottom curve is circular polarization for the same model atmosphere, while the error bars represent measurements of circular polarization from Angel *et al.* (1974). The horizontal bars specify the wavelength intervals over which the broad band measurements were taken. For the values of circular polarization, the scale on the right has units of percentage, with positive polarization values upward.

scattering and an unaltered temperature profile, or a combination of these effects, could approximately reproduce the spectrum of G240 – 72.) The long depression in the theoretical results blueward of 5500 Å could explain Greenstein's (1974) finding that the spectrum of G240 – 72 cannot be fitted by a blackbody spectrum: a fit at $\lambda > 5500$ Å leaves an ultraviolet excess.

The use of a value of $\nu \sim 10^{11}$–10^{12}/s is not unreasonable for a cool ($T_e \approx 6000$ K) white dwarf, if the cyclotron transition broadens in a similar manner to any other bound–bound transition. For van der Waals broadening due to hydrogen, the C_6 coefficient can be estimated from Unsold (1955) to be $\sim 7 \times 10^{-32}$ where we have used for the radius of the electron's wave function, the radius $r = 2$ Å of the first excited Landau level (Liebert *et al.* 1978). At $T = 6000$ K, this translates into a collision frequency $\nu \approx 10^{-8} n_H$/s, where n_H is the number density of neutral hydrogen. For the $T_e = 6000$ K, $\log g = 8.0$, hydrogen rich and metal deficient model atmosphere used in our computations, we find n_H ranges roughly from 10^{18} to 10^{20} as τ goes from roughly 10^{-3} to 1. A reduction in the hydrogen abundance by a factor of $\sim 10^2$ would suffice to bring ν to a value of $\sim 10^{12}$/s at low optical depths, since similar expressions would hold for van der Waals broadening by helium.

Comparison with observations of polarization is perhaps a more telling test for any proposed model invoking cyclotron absorption. Tables 2 and 3 give values of linear and circular polarization for the models discussed earlier and whose flux reductions are listed in Table 1.

In the case of G240 – 72, Angel *et al.* (1974) found negative circular polarization of about –0.5 per cent between 4000 Å and 5000 Å, and a positive value approaching 1 per cent between 5500 Å and 7000 Å. Our theoretical calculations for collisionally broadened cyclotron absorption do indeed give a change in the sign of circular polarization. This results from the dipole field geometry, which gives $\cos \psi$ as positive and negative on different portions of the disc; the net circular polarization then depends on which portions of the disc have absorption in the core of the cyclotron feature and which in the wings. Clearly the

Table 2. Values of linear polarization in percentage due to cyclotron absorption for a number of cases. Unless otherwise indicated, $B_d = 3.8 \times 10^8$ G, $i = 75°$, $\nu = 5 \times 10^{11}$/s, T_e = 6000 K, T_s = 4881 K, $f_U = 0$.

Case	Wavelength 4000Å	4500Å	5000Å	5500Å	6000Å
$B_d=0$	0	0	0	0	0
$\nu=0$	-0.004	-0.001	0.006	0.16	0
$\nu=5\times10^{11}s^{-1}$	-0.9	0.01	2.0	5.9	0.6
$\nu=5\times10^{12}s^{-1}$	1.5	3.1	6.6	8.2	3.9
$\nu=5\times10^{10}s^{-1}$	-0.5	-0.1	0.5	2.4	0.07
Cyclotron strength increased by factor of 10	1.5	3.1	6.6	8.2	3.9
$i=0^\circ$	0	0	0	0	0
$i=45^\circ$	-0.6	0.9	1.7	2.2	0.3
$i=90^\circ$	-1.0	-0.1	1.9	6.7	0.7
$B_d=2\times10^8$G, $f_U=0.9$	1.1	1.3	3.3	31.4	2.7
$T_s=4000$K	-1.2	0.2	3.1	8.7	1.0
$T_s=3000$K	5.0	4.6	8.8	12.9	3.4
Continuum polarised, $\nu=0$	1.1	0.9	0.9	0.8	0.5
Continuum polarised, $\nu=5\times10^{11}s^{-1}$	0.4	1.1	2.6	6.1	1.0

Table 3. Values of circular polarization in percentage due to cyclotron absorption for a number of cases. Unless otherwise indicated, $B_d = 3.8 \times 10^8$ G, $i = 75°$, $\nu = 5 \times 10^{11}$/s, T_e = 6000 K, T_s = 4881 K, $f_U = 0$.

Case	Wavelength 4000Å	4500Å	5000Å	5500Å	6000Å
$B_d=0$	0	0	0	0	0
$\nu=0$	-0.01	-0.02	0.06	0.2	0
$\nu=5\times10^{11}s^{-1}$	2.3	-0.8	0.6	2.9	0.3
$\nu=5\times10^{12}s^{-1}$	-4.9	-1.5	1.6	3.5	2.0
$\nu=5\times10^{10}s^{-1}$	-0.8	-0.3	0.15	1.2	0.04
Cyclotron strength increased by factor of 10	-4.9	-1.5	1.6	3.5	2.0
$i=0^\circ$	-0.5	2.2	2.1	0.9	0.2
$i=45^\circ$	-4.3	-0.8	1.8	4.6	0.6
$i=90^\circ$	0	0	0	0	0
$B_d=2\times10^8$G, $f_U=0.9$	-0.6	-0.8	-1.9	-18.3	-1.6
$T_s=4000$K	-3.4	-1.0	1.0	4.4	0.6
$T_s=3000$K	-7.3	-1.7	2.9	6.3	1.9
Continuum polarised, $\nu=0$	-3.0	-2.8	-2.5	-2.3	-2.3
Continuum polarised, $\nu=5\times10^{11}s^{-1}$	-4.6	-3.4	-2.3	-0.5	-2.1

details of the polarization spectrum will depend rather critically on the field geometry. The observation of a value of circular polarization of about 0.4 per cent near 3500 Å is harder to explain using cyclotron absorption, but may be accounted for a combination of polarization in the continuum and the tailing off of cyclotron absorption at low wavelengths.

In addition, the positive value of circular polarization around 3500 Å may be due to the influence of the first harmonic of the collisionally broadened cyclotron absorption. We have not included this absorption effect because of lack of knowledge of its strength, but its location is suitable for increasing the circular polarization near 3500 Å. Finally, there is the problem of the generally small observed values of circular polarization in G240 – 72 (Angel *et al.* 1974). To obtain such small values requires a viewing angle $i \gtrsim 85^\circ$, which *a priori* may be considered unlikely. Again though, if birefringence or scattering are important, the degree of circular polarization would be reduced and the appropriate viewing angle correspondingly increased. In our models we have assumed that these effects are negligible, and hence in Fig. 2 present theoretical values of flux and circular polarization at $i = 85^\circ$ for comparison with observations of G240 – 72.

Finally, to suggest the possible impact of continuum polarization on our results, we include in Tables 2 and 3 results for $\nu = 0$ and $\nu = 5 \times 10^{11}$/s in which Lamb & Sutherland's (1974) continuum shifts are used except that no shifts are made blueward of 3000 Å or redward of 8000 Å. The most that these results can show is the limitations of the calculations for polarization, since the actual effect of the shift could conceivably lead to quite a number of different effects on the results. Hence due to the lack of suitable theory for the polarization of the continuum, our analysis of polarization for G240 – 72 must be considered tentative.

Another star for which cyclotron absorption has been considered as a possible interpretation of observed features is LP790 – 29 (Liebert *et al.* 1978). The difficulty of explaining the observed polarization and the fine structure within the observed absorption band in terms of cyclotron absorption has been emphasized by Liebert *et al.* (1978). In addition it is clear from the present calculations that collisionally broadened cyclotron absorption cannot explain the observed width (~ 1500 Å) and central depth (~ 80 per cent) even if allowance is made for a high degree of field uniformity or birefringence. The alternative interpretation in terms of C_2 absorption would appear much more reasonable for this star (Liebert *et al.* 1978; Wickramasinghe & Bessell 1979).

4 Conclusions

The presence of extremely high magnetic fields ($\gtrsim 10^8$ G) can give rise to cyclotron absorption in white dwarfs. Doppler broadened absorption is strong, but so narrow that in normal circumstances it results in only trivial spectral features. Collisionally broadened absorption is also very strong. For low collision rates it is also very narrow, while for high collision rates it results in a broad shallow depression of the spectrum rather than a dramatic feature.

There are several ways in which collisionally broadened cyclotron absorption may give rise to more significant absorption features: through absorption of both left and right handed circularly polarized light made possible through birefringence or scattering, which can deepen the absorption feature by up to a factor of 2; through a very uniform magnetic field, which results in a deep but relatively narrow feature; and through a greatly increased temperature gradient in the surface layer of the stellar atmosphere, which results in a much deeper but still very broad absorption feature. A combination of these latter processes may explain the broad absorption feature found in white dwarf G240 – 72.

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References

Angel, J. R. P., 1977. *Astrophys. J.,* **216**, 1.

Angel, J. R. P., 1978. *Ann. Rev. astr. Astrophys.,* **16**, 487.

Angel, J. R. P., Hintzen, P., Strittmatter, P. A. & Martin, P. G., 1974. *Astrophys. J.,* **190**, L71.

Bekefi, B., 1966. *Radiation Processes in Plasmas,* John Wiley, New York.

Greenstein, J. L., 1974. *Astrophys. J.,* **194**, L51.

Kemp, J. C., 1970. *Astrophys. J.,* **162**, 169.

Lamb, F. K. & Sutherland, P. G., 1971. Paper presented at the *Conference on Line Formation in the Presence of Magnetic Fields,* 1971 August 30–September 2, High Altitude Observatory, Boulder, Colorado.

Lamb, F. K. & Sutherland, P. G., 1974. *Physics of Dense Matter,* p. 265, ed. Hansen, C. J., D. Reidel, Dordrecht, Holland.

Liebert, J., 1976. *Publs astr. Soc. Pacif.,* **88**, 490.

Liebert, J., Angel, J. R. P. & Stockman, H. S., 1978. *Astrophys. J.,* **225**, 181.

Martin, B. & Wickramasinghe, D. T., 1978. *Mon. Not. R. astr. Soc.,* **183**, 533.

Martin, B. & Wickramasinghe, D. T., 1979. *Mon. Not. R. astr. Soc.,* in press.

Unsöld, A., 1955. *Physik der Sternatmosphären,* Springer, Berlin.

Wickramasinghe, D. T. & Bessell, M. S., 1979. *Mon. Not. R. astr. Soc.,* **188**, 841.

Wickramasinghe, D. T., Cottrell, P. L. C. & Bessell, M. S., 1977. *Astrophys. J.,* **217**, L65.

Wickramasinghe, D. T. & Martin, B., 1978. *Proc. astr. Soc. Aust.,* **3**, 269.