Methods for Calculating Circular Polarisation in Magnetic White Dwarfs

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The problem of radiative transfer in the presence of a magnetic field may be formulated in terms of four interdependent differential equations for the four Stokes parameters *I, Q, U and V* (Hardorp *et al.* 1976):

$$\mu \frac{dI}{d\tau} = \eta_I(I - B) + \eta_Q Q \qquad + \eta_V V, \quad (1)$$

$$\mu \frac{dQ}{d\tau} = \eta_Q(I - B) + \eta_I Q - \rho_R U, \quad (2)$$

$$\mu \frac{dU}{d\tau} = \qquad \rho_R Q + \eta_I U - \rho_W V, \quad (3)$$

$$\mu \frac{dV}{d\tau} = \eta_V(I - B) \qquad + \rho_W U + \eta_I V. \quad (4)$$

where

$$\eta_I = \tfrac{1}{2}\eta_p \sin^2\psi + \tfrac{1}{4}(\eta_\ell + \eta_r)(1 + \cos^2\psi), \quad (5)$$

$$\eta_Q = (\tfrac{1}{2}\eta_p - \tfrac{1}{4}(\eta_\ell + \eta_r)\sin^2\psi, \quad (6)$$

$$\eta_V = \tfrac{1}{2}(\eta_r - \eta_\ell)\cos\psi, \quad (7)$$

where $\mu = \cos\theta$, θ is the angle between the propagation direction and the normal to the surface of the star, ψ is the angle between the propagation direction and the direction of the local magnetic field, B is the local source function, η_p, η_r and η_ℓ are the ratios of the total absorption coefficient of the three shifted Zeeman components to the unpolarised continuum absorption coefficient and ρ_R and ρ_W are terms introducing anomalous dispersion (Wittmann 1974).

The basic physical system being modelled in the absence of a magnetic field is the standard equation

$$\mu \frac{dI}{d\tau} = \eta(I - B) \ , \quad (8)$$

that is, the emitted intensity is due to the source function $B(\tau)$ at various optical depths τ, with absorption at each lesser depth according to the coefficient η. All this takes place in an LTE, one-dimensional, classical transfer model with negligible scattering. The decomposition of η in (8) into η_I, η_Q and η_V in (5)-(7) introduces polarisation of the emerging beam: the linear polarisation is given by $(Q^2 + U^2)^{1/2}/I$ and the circular polarisation by V/I. The anomalous dispersion terms ρ_R and ρ_W cause mixing into component U; when $\rho_R = 0$ and $\rho_W = 0$, $U = 0$ and the system reduces to three equations.

In Martin & Wickramasinghe (1979) an analytical solution is presented to (1)-(4) based on a stepwise method in τ using an Unno (1956) type solution in each interval (τ_i, τ_{i+1}), $i = 1, \ldots, N$. In that paper the results obtained using the analytical solution are compared with results obtained using several other methods proposed for direct solution of (1)-(4): an Unno (1956) solution using an approximate linear source function, the method of Shipman (1971) which gives expressions for I and V based partly on physical reasoning, a direct Runge-Kutta numerical solution (Beckers 1969) and a perturbation method (Hardorp *et al.* 1976). Compared to the analytical method, each of these has limitations in accuracy (Unno, Shipman), computation speed (Runge-Kutta) or to a lesser extent both (Hardorp *et al.*).

In this paper we compare the analytical method with two approximate methods which give circular polarisation results directly for flux rather than intensity. Each of these methods has been used in interpreting data from magnetic white dwarfs.

The method of Landstreet & Angel (1975) — see also Angel & Landstreet (1974) — is based on the Shipman method but also involves an approximate method for the optical depth integration assuming a grey atmosphere. The result for circular polarisation is

$$V/I = -\frac{h\nu}{8kT}(1 - \exp(-h\nu/kT))^{-1} \frac{\varkappa/\varkappa_\nu}{1 + \varkappa/\varkappa_\nu} \frac{\triangle\varkappa}{\nu} \cos(i), \quad (9)$$

where i is the angle between the direction of the magnetic dipole and the line of sight to the observer, $\varkappa$ is the standard opacity and $\varkappa_\nu$ the opacity at frequency ν each evaluated at $\tau = 2/3$ and

$$\frac{\triangle\varkappa}{\varkappa} = \beta \frac{\nu_L}{\nu} \quad (10)$$

where $\nu_L = e\overline{B}_z/4\pi mc$ is an averaged Larmor frequency and β is the circular dichroism parameter which depends on the type of opacity sources.

The method of Brown *et al.* (1977) is also based on the treatment by Shipman (1971). Their approximation for circular polarisation is

$$V/I = (F' - F)/(F' + F) \quad (11)$$

where F is the flux obtained solving (8) using the unpolarised opacities (η in (8), $\varkappa_\nu/\varkappa$ in (9)) and F' the flux obtained using the same opacities multiplied by the factor $(1 + \triangle\varkappa/\varkappa)$. Brown *et al.* give $\beta = 8$ for a hydrogen atmosphere, and

$$\overline{B}_z = \frac{B_p \cos(i)}{20} \ \frac{15 + u}{3 - u} \quad (12)$$

where B_p is the polar field strength and $u \sim 0.35$.

In Table 1 we present results for the case $B_p = 9 \times 10^6$G and $i = 0°$, 30° and 60°, using for the analytical method and for the method of Brown *et al.* the $T_e = 12000$K atmosphere of Wickramasinghe (1972).

Although Martin & Wickramasinghe (1979) found that the method of Shipman (1971) underestimated the circular polarisation by nearly a factor of two in a real atmosphere, the results in Table 1 show that the approximate flux-based methods of Landstreet and Angel (1975) and Brown *et al.* (1977) are accurate to within 10% to 20% for the continuum wavelengths 4000Å and 5500Å and for the continuum interval

Table I. Values for circular polarisation using the analytical method (Martin & Wickramasinghe 1979), the method of Landstreet & Angel (1975) and the method of Brown *et al.* (1977) for a magnetic white dwarf, $T_e = 12000$K, with temperature and opacity structure given by Wickramasinghe (1972), dipole magnetic field with polar field strength $B_p = 9\times10^6$G, viewed at an angle i with respect to the dipole field direction. The values in parentheses are given by Brown *et al.* (1977): note that the calculational procedure for obtaining these is different in a number of details from that used in this paper to obtain the unparenthesised values listed for Landstreet & Angel and for Brown *et al.*

$i=0°$ (Å)	*Analytical* (%)	*Landstreet & Angel* (%)	*Brown et al.* (%)
λ = 4000	−0.82	−0.99	−0.90
λ = 5500	−0.74	−0.79	−0.84
λλ3710-5500	−0.79	−0.92	−0.88
λλ3710-5500 with lines	−1.01		
λλ3500-5500	−0.11	−0.90	−0.87
λλ3500-5500 with lines	−0.25		
$i=30°$			
λ = 4000	−0.71	−0.86	−0.78
λ = 5500	−0.64	−0.69	−0.73
λλ3710-5500	−0.68	−0.80	−0.76
λλ3710-5500 with lines	−0.89		
λλ3500-5500	−0.10	−0.78	−0.76
λλ3500-5500 with lines	−0.24		
$i=60°$			
λ = 4000	−0.40	−0.50	−0.45
λ = 5500	−0.37	−0.40	−0.43
λλ3710-5500	−0.39	−0.46	−0.44
λλ3710-5500 with lines	−0.52	(−0.42)	(−0.58±0.15)
λλ3500-5500	−0.06	−0.45	−0.44
λλ3500-5500 with lines	−0.15		(−0.39±0.15)

λλ3710-5500 Å. The apparent reason for the success of the methods is that when calculating v with the method of Shipman, the results are not too far off as long as Q is very small, which will be the case when $\eta_Q << \eta_V$. In the case of continuum polarisation and sufficiently small fields we have $\eta_r \sim \eta_p + \eta'$ and $\eta_\ell \sim \eta_p - \eta'$ (Lamb & Sutherland 1974) so that indeed $\eta_Q << \eta_V$.

In an isolated absorption line on the other hand, a typical configuration is $\eta_r = \alpha\eta_\ell = \alpha\eta_p$ where α is in the range 1.1 to 10^6, hence $\eta_Q \sim \eta_V$ and methods based on Shipman's approach cannot be expected to give useful results at individual wavelengths. In averaging over a number of wavelengths and hence over a number of lines which cause circular polarisation of opposite senses, the net effect of the lines may not be too great, as seems to be the case from the results in Table 1 for the interval λλ3710-5500 Å when lines are included. However, the Balmer absorption edge causes a drastic change in average circular polarisation, and the approximate methods do not succeed in reproducing this. Brown *et al.* (1977) include an ad hoc treatment of the Balmer edge but obviously this is not sufficient to overcome the intrinsic limitations of their approximate method away from the uninterrupted portion of the continuum.

The sizable discrepancy between their theoretical value $V/I = (-0.39\pm0.15)$% for λλ3500-5500 Å and $i=60°$ and the observed values for magnetic DA white dwarf GD90 of (-0.12 ± 0.08)% (Brown *et al.* 1977) and (-0.03 ± 0.15)% (Angel *et al.* 1974) was pointed out by Wickramasinghe & Martin (1979). On the basis of the analysis and results here, this discrepancy is in large part due to the inadequacies of Brown *et al.*'s method for calculating V.

In conclusion, the approximate methods of Landstreet & Angel (1975) and Brown *et al.*'s method for calculating V.

In conclusion, the approximate methods of Landstreet & Angel (1975) and Brown *et al.* (1977) usually should give a rough approximation to the circular polarisation in the continuum, but in the presence of absorption lines or absorption edges these methods cannot be relied upon. Since an accurate and easy-to-programme analytical solution (Martin & Wickramasinghe 1979) is now available, it should henceforth be unnecessary to resort to the approximate methods.

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