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Magneto-optical effects in magnetic white dwarfs – I. The line spectra

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Summary. The impact of magnetic birefringence or magneto-optical effects on the line spectra of magnetic white dwarfs is investigated using model calculations. It is shown that at fields at which the line splitting resembles that of an overlapping Zeeman triplet, the central depth of the *p* component can be increased by as much as 50 per cent. Changes are also found in polarization near the central component.

1 Introduction

In a recent series of papers, we have attempted to model the spectra of magnetic white dwarfs under various simplifying assumptions (Martin & Wickramasinghe 1978; Wickramasinghe & Martin 1979; see also the confirmatory study of O'Donoghue 1980). We have shown that in most cases, a centred or slightly off-centred dipole field distribution gives results in reasonable agreement with observations of magnetic white dwarfs. However, full detailed agreement is lacking. The ultimate expectation is that with the development of more sophisticated models, it will be possible to determine unambiguously the surface field distribution from an analysis of spectroscopic and spectropolarimetric data. One important simplification in our models has been the neglect of magneto-optical effects in the radiative transfer problem. In this paper we investigate the conditions under which magneto-optical effects can have a significant influence on the line intensity and polarization spectra of magnetic white dwarfs.

The equations describing the transfer of polarized light in the presence of a magnetic field are written normally in terms of the four Stokes parameters I, Q, U and V (Hardorp, Shore & Wittmann 1976):

$$\mu \frac{dI}{d\tau} = \eta_I(I-B) + \eta_Q Q + \eta_V V, \qquad (1)$$

$$\mu \frac{dQ}{d\tau} = \eta_Q(I-B) + \eta_I Q - \rho_R U, \qquad (2)$$

$$\mu \frac{dU}{d\tau} = \rho_R Q + \eta_I U - \rho_W V, \tag{3}$$

$$\mu \frac{dV}{d\tau} = \eta_V (I - B) + \rho_W U + \eta_I V. \tag{4}$$

Here τ is the optical depth, $\mu = \cos\theta$, where θ is the angle between the propagation direction and the axis along which τ is measured, B is the local source function and

$$\eta_I = \tfrac{1}{2}\eta_p \sin^2\psi + \tfrac{1}{4}(\eta_l + \eta_r)(1 + \cos^2\psi), \tag{5}$$

$$\eta_Q = [\tfrac{1}{2}\eta_p - \tfrac{1}{4}(\eta_l + \eta_r)] \sin^2\psi, \tag{6}$$

$$\eta_V = \tfrac{1}{2}(\eta_r - \eta_l)\cos\psi, \tag{7}$$

where ψ is the angle between the propagation direction and the direction of the magnetic field and η_p, η_l and η_r are the ratios of the total absorption coefficient of the three shifted Zeeman components plus the shifted continuum absorption coefficient to the (unshifted) continuum absorption coefficient. The solution pair

$$\begin{pmatrix} Q \\ U \end{pmatrix}$$

to equations (1)–(4) must be multiplied by

$$\begin{pmatrix} \cos 2\phi & -\sin 2\phi \\ \sin 2\phi & \cos 2\phi \end{pmatrix},$$

where ϕ is the azimuth with respect to an arbitrary x-axis at right angles to the line-of-sight.

The effect of the magnetic field is to make η_r and η_l different from η_p, hence to make η_Q and η_V non-zero and so produce non-zero linear polarization of magnitude $(Q^2 + U^2)^{1/2}/I$, circular polarization ($= V/I$) and the normal Zeeman triplet splitting of absorption lines in the intensity I.

The terms ρ_R and ρ_W introduce magneto-optical effects. ρ_R leads to a rotation of the electric vector of linearly polarized light, while ρ_W leads to a phase retardation between linear polarizations which are parallel and perpendicular to the magnetic field (Wittmann 1974). Terminology is listed in Table 1.

Table 1. Terms used in referring to magneto-optical effects.

General
Magneto-optical effects
Magnetic optical birefringence
Elliptical birefringence
Anomalous dispersion
ρ_R only (longitudinal magnetic field: $\psi = 0$, therefore $\rho_W = 0$)
Faraday effect or Faraday rotation
Induced circular birefringence
Macaluso–Corbino effect
ρ_W only (transverse magnetic field: $\psi = \pi/2$, therefore $\rho_R = 0$)
Voigt effect
Induced linear birefringence
Cotton–Mouton effect

Equations (1)–(4) have been studied by a number of authors both in the absence of magneto-optical effects (Unno 1956) and in their presence (Kai 1968; Beckers 1969; Stenflo 1971; Wittmann 1974, 1977; Hardorp *et al.* 1976). Most of this work has dealt with either the special, mathematically tractable case of a source function B linear in optical depth τ or with specific illustrative cases. In this paper we investigate the impact of magneto-optical effects on the emergent line spectrum of realistic models of magnetic white dwarfs. Throughout we use the analytical solution to equations (1)–(4) presented by Martin & Wickramasinghe (1979).

2 Theory

To incorporate magneto-optical effects in lines, it is easiest to start with a Voigt line profile:

$$H(a, v) = \frac{a}{\pi} \int_{-\infty}^{\infty} \frac{\exp[-(y+v)^2]}{y^2 + a^2}\, dy. \tag{8}$$

Here $a = \gamma\lambda^2/4\pi c\Delta\lambda_D$ is the damping parameter, $v = \Delta\lambda/\Delta\lambda_D$ is the position within the profile and $\Delta\lambda_D$ is the Doppler width. ρ_R and ρ_W in the presence of such a component of a line and a magnetic field are given by

$$\rho_R = -\eta_0(F_r - F_l)\cos\psi, \tag{9}$$

$$\rho_W = -\eta_0[F_p - \tfrac{1}{2}(F_l + F_r)]\sin^2\psi, \tag{10}$$

where η_0 is the value of η_p (or η_l or η_r) when $a = v = 0$ and

$$F(a, v) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{y\exp[-(y-v)^2]}{y^2 + a^2}\, dy \tag{11}$$

is the dispersion function, $F_p = F(a, v - v_p)$, $F_l = F(a, v - v_l)$ and $F_r = F(a, v - v_r)$ where v_p, v_l and v_r are the amounts by which components p, l and r are shifted in the ambient magnetic field. As in the case of η_I, η_Q and η_V, the values of ρ_R and ρ_W in the presence of an absorption line are obtained by summing over the different line components using appropriate strengths and weighting factors and using appropriate shifts v_p, v_l and v_r for each component.

$F(a, v)$ clearly is closely related to $H(a, v)$:

$$F(a, v) = a^{-1}\left[\tfrac{1}{2}vH(a, v) + \tfrac{1}{4}\frac{dH(a, v)}{dv}\right]. \tag{12}$$

Note that ρ_R and ρ_W are analogous to η_Q and η_V, respectively, with $\eta_0 F$ replacing H and a reversal of signs. However, while H is symmetric in v, giving rise to the familiar symmetric line profiles, $F(a, -v) = -F(a, v)$ and hence ρ_R and ρ_W can give rise to non-symmetric effects.

In the literature there are some differences in nomenclature, as noted in Table 2. In all cases the red component of absorption has $\Delta m = m_{\text{upper}} - m_{\text{lower}} = -1$ and is calculated using $H(a, v - v_i)$ and $F(a, v - v_i)$ where $i = p$, l and r. In emission the polarization is such that the **E** vector at a fixed point in space rotates clockwise as the electromagnetic wave travels in the same direction as the **B** vector. This is called in classical optics right circularly polarized (RCP) light, as is done by Hardorp *et al.* (1976), but is called left circularly polarized (LCP) light by various astrophysicists. (We have adopted the later notation,

Table 2. Notations used in referring to red and blue Zeeman components.

Author	Component $\Delta m = +1$, blue	$\Delta m = -1$, red	Corrections
Unno (1956)	*r* (right)	*l* (left)	
Beckers (1969)	b (blue)	r (red)	ρ_R and ρ_W too large by factor $2H(a, v)$
Stenflo (1971)	b (blue)	r (red)	
Wittman (1974, 1977)	*r* (right)	*l* (left)	Signs of ρ_R and ρ_W incorrect
Hardorp *et al.* (1976)	*l* (left)	*r* (right)	
Martin & Wickramasinghe (1978, 1979)	*r* (right)	*l* (left)	

consistent with our previous publications.) The blue component of course is associated with opposite signs and rotations.

3 Results

It is straightforward to make the calculations of line absorption with and without ρ_R and ρ_W. However, since understanding the nature of the effect is not so easy, we first use a simple case to illustrate how the effects arise. In Fig. 1 are H and F in a hypothetical isolated

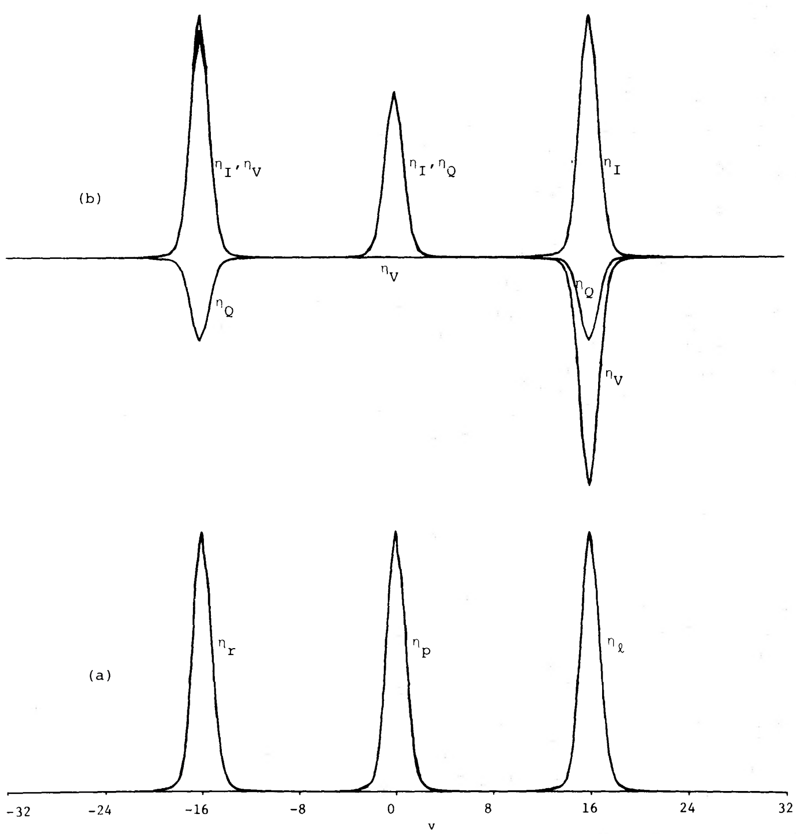

Figure 1. Parameter values for a hypothetical Zeeman triplet with $\lambda(v = 0) = 5000$ Å, $T-\tau$ relation taken from the zero-field, high-gravity ($\log g = 8.0$) model atmosphere for a DA white dwarf with $T_e = 20\,000$ K given by Wickramasinghe (1972), continuum opacities $\eta_p = \eta_l = \eta_r = 4$ independent of optical depth, $\eta_0 = 10\,000$, $v_r = -16$, $v_p = 0$, $v_l = 16$, $a = 0.1$, $\mu = 0.8$, $\cos\psi = 0.7$ and $\cos 2\phi = 0.6$. Illustrated are: (a) η_r, η_p, and η_l; (b) η_I, η_Q and η_V; (c) F_r, F_p and F_l; (d) ρ_R and ρ_W; (e) intensity I calculated without ρ_R and ρ_W (curve 3) and with ρ_R and ρ_W (curve 4); (f) linear polarization without and with ρ_R and ρ_W (curves 3 and 4 respectively); (g) circular polarization without and with ρ_R and ρ_W (curves 3 and 4 respectively). The normalizations are arbitrary.

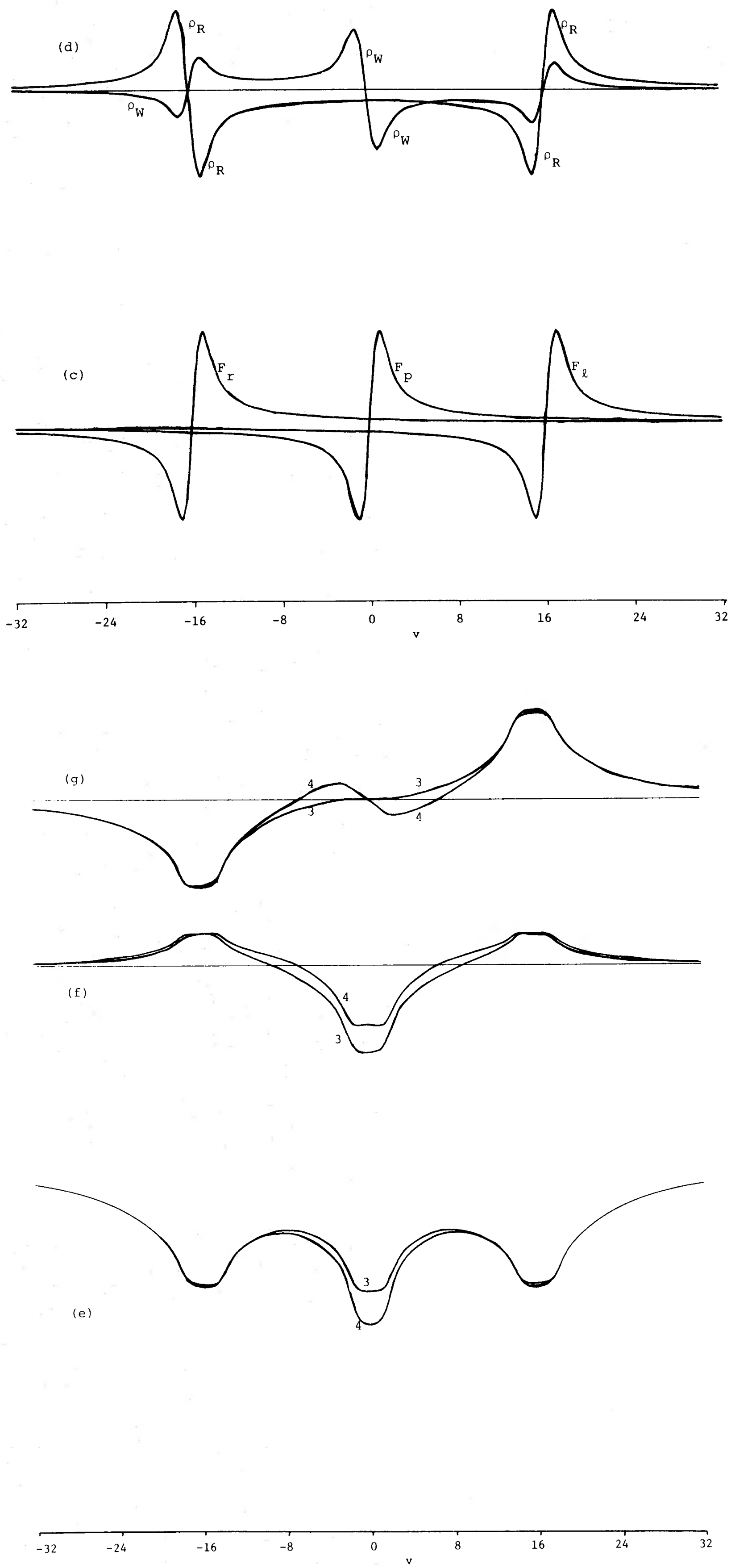

Figure 1 – *continued*

Zeeman triplet with η_I, η_Q, η_V, ρ_R, ρ_W and the resultant intensity, linear and circular polarizations illustrated. The solutions without ρ_R and ρ_W can be understood qualitatively on the basis of the Unno solution:

$$I_U = B_0\left(1 + \frac{\beta\mu\eta_I}{\eta_I^2 - \eta_Q^2 - \eta_V^2}\right), \tag{13}$$

$$Q_U = -B_0 \frac{\beta\mu\eta_Q}{\eta_I^2 - \eta_Q^2 - \eta_V^2}, \tag{14}$$

$$V_U = -B_0 \frac{\beta\mu\eta_V}{\eta_I^2 - \eta_Q^2 - \eta_V^2}, \tag{15}$$

where $B = B_0(1 + \beta\tau)$. Although not correct for the non-Unno T–τ profile used here, equations (13)–(15) give the general character of the solution. The main effect of ρ_R and ρ_W is in the region of the p-component (near $v = 0$) of the triplet, where the emitted intensity is lowered, the magnitude of linear polarization is reduced and positive and negative bulges in circular polarization are introduced. These effects can be explained in general mathematical terms as follows.

For the central, p-component, $\eta_V = 0$. Hence when $\rho_R = \rho_W = 0$, equations (1)–(4) show that absorption will only affect I and Q. When ρ_R and ρ_W are introduced as in Fig. 1(d), U becomes negative through the term $\rho_R Q$, Q is reduced in magnitude (though still remaining negative) through the term $-\rho_R U$, and I is reduced through the term $\eta_Q Q$; other interactions of course contribute to the final result. The net effect is to decrease the magnitude of linear polarization $[(Q^2 + U^2)^{1/2}/I]$ and reduce I. On either side of the centre of the p-component, the non-symmetric term $\rho_W U$ causes the bulges illustrated in Fig. 1(g).

For the red component, $\eta_V \neq 0$ and $\eta_Q \neq 0$ so that Q and V are non-zero even when $\rho_R = \rho_W = 0$. When ρ_R and ρ_W become non-zero, there is little change in the solution because U remains very small compared with I, Q and V. To see this, first note that the solution ratio Q/V at any depth approximately equals η_Q/η_V, as is apparent from equations (14) and (15). Second, note that according to equations (6), (7), (9) and (10), if $F \propto H$, then $\rho_R/\eta_V = \rho_W/\eta_Q$. Equation (12) shows that for fixed v, $F \propto H$ may not be a bad approximation away from the centre ($v = 0$) of a component giving rise to H and F, so that in equation (3), $\rho_R Q - \rho_W V \ll \eta_Q I$ or $\eta_V I$ in the r-component, therefore $U \ll Q$ or V and hence ρ_R and ρ_W have little effect on the solution. The same considerations apply to the blue component. Note also from Fig. 1(d), (f) and (g) that the signs of ρ_R, ρ_W, Q and V are always such that $U \ll Q$ or V in both red and blue components.

We have just described some general mathematical reasons why the p-component primarily is affected by ρ_R and ρ_W from adjacent p- and r-components or from adjacent p- and l-components. We have also found that there is no noticeable effect due to ρ_R and ρ_W from adjacent r- and l-components.

The results illustrated in Fig. 1 show an impact on the spectrum of a magnetic white dwarf due to magneto-optical effects as large as any which is likely to occur. Our results indicate that when the field is stronger and the components are more widely separated than in Fig. 1, ρ_R and ρ_W from each component are smaller in the region of the other components and the effect on the spectrum is smaller. But when the field is weaker and the components are closer together, ρ_R and ρ_W as well as η_Q and η_V are smaller in magnitude compare with η_I – see equations (9), (10), (6) and (7) – and this again reduces the influence of ρ_R and ρ_W on the result. For zero field, $F_r = F_l = F_p$ and therefore $\rho_R = \rho_W = 0$.

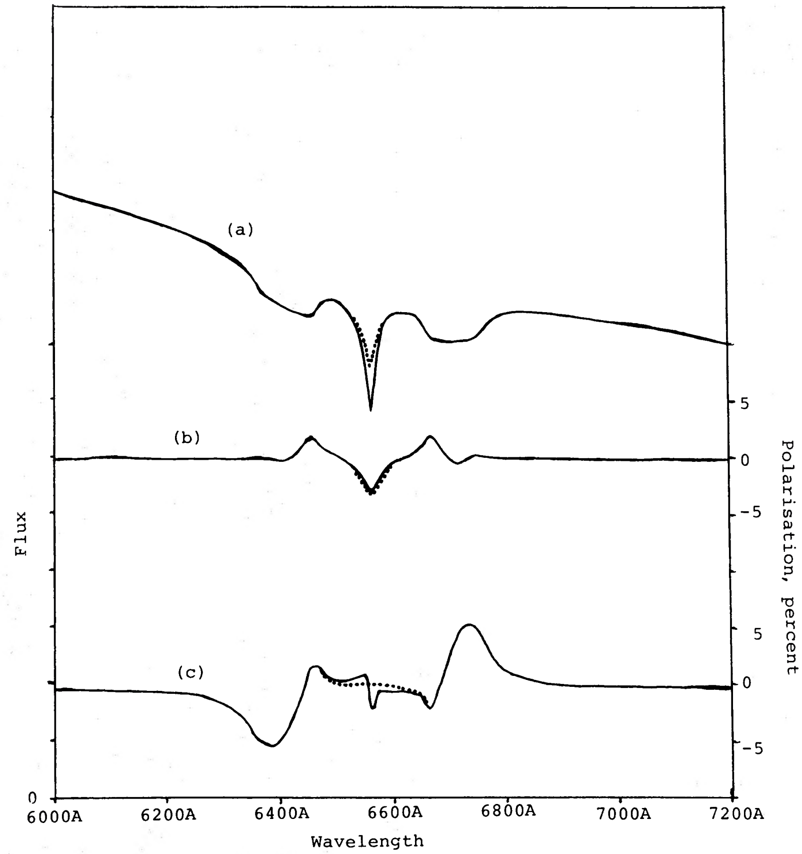

Figure 2. Flux and polarizations for a realistic model magnetic white dwarf near the Hα absorption line, calculated with and without the inclusion of the parameters ρ_R and ρ_W which introduce magneto-optical effects. The white dwarf model atmosphere is the zero-field, high-gravity ($\log g = 8.0$), T_e = 20 000 K model taken from Wickramasinghe (1972). The strength of the assumed dipole magnetic field is taken to be $B_d = 10^7$ G. The star is viewed at an angle $i = 45°$ with respect to the dipole axis. Illustrated are: (a) flux on a linear scale without ρ_R and ρ_W (dotted line) and with ρ_R and ρ_W (continuous line); (b) linear polarization without and with ρ_R and ρ_W (dotted and continuous, respectively); (c) circular polarization without and with ρ_R and ρ_W (dotted and continuous, respectively).

In Fig. 2 we present an illustrative case of the impact of magneto-optical effects on the spectrum of a magnetic white dwarf star calculated using a realistic model. The model atmosphere is again taken from Wickramasinghe (1972), but in this case results are given for flux and polarizations averaged over the surface of the star assuming a magnetic dipole field distribution, using realistic opacities varying with optical depth, the full set of components for the hydrogen absorption line Hα using shifts v_p, v_l and v_r from Kemic (1974), and polarization of the continuum (Lamb & Sutherland 1974).

The results in Fig. 2 should only be used to indicate the likely impact of magneto-optical effects in the absorption lines of magnetic white dwarfs. In particular, we have not attempted here to produce accurate values for polarization in the extreme line wings, both because the Voigt line profile used here does not adequately represent real absorption wings and because magneto-optical effects in the continuum will be important in the far wings. We plan to treat the latter effect in a following paper.

For Hα which resembles an overlapping Zeeman triplet at the fields used to calculate Fig. 2, the qualitative effects are quite similar to those obtained in the simplified model illustrated in Fig. 1. The main effects are a deepening of the central component, a slight reduction in the magnitude of linear polarization and the introduction of antisymmetric

Table 3. Linear and circular polarization across Hα.

λ (Å)	Linear polarization (without ρs)	Circular polarization (without ρs)	Linear polarization (with ρs)	Circular polarization (with ρs)
6000	0.000011	−0.0038	0.000054	−0.0038
6100	0.000054	−0.0057	0.000094	−0.0050
6200	0.00042	−0.0069	0.00059	−0.0064
6300	0.00061	−0.0159	0.000131	−0.0164
6400	−0.0037	−0.051	−0.0042	−0.050
6500	0.00075	0.00023	0.0026	0.0057
6600	−0.0058	−0.0028	−0.0038	−0.0066
6700	0.00133	0.035	−0.00023	0.032
6800	0.00070	0.0103	0.00025	0.0109
6900	0.000099	−0.00065	−0.000013	−0.00030
7000	0.000131	−0.0024	−0.0000088	−0.0018
7100	0.000028	−0.0036	−0.000051	−0.0035
7200	0.000024	−0.0037	0.000025	−0.0037

bulges in circular polarization near the *p*-component. The changes in polarization caused by the inclusion of magneto-optical effects can be judged by the results presented in Table 3. It is interesting to note that a qualitatively similar behaviour by circular polarization across a triplet occurs even in the absence of magneto-optical effects if one assumes an off-centred dipole field distribution (Wickramasinghe & Martin 1979). It is accordingly important to include magneto-optical effects when determining information on the field structure from polarization data.

We find that when the Zeeman components become widely separated, magneto-optical effects are not important. However, in most high field cases, there is a significant overlapping between Zeeman components from different lines. In these circumstances, the depths of the *p*-components could be significantly increased by magneto-optical effects.

4 Conclusion

We conclude that magneto-optical effects can significantly alter both the polarization and intensity spectra of magnetic white dwarfs. When the line splitting resembles that of an overlapping triplet, we have shown that the central depth of the *p*-component can increase by as much as 50 per cent. Antisymmetric bulges in circular polarization are introduced near the *p*-component, an effect which could be misinterpreted as due to field geometry.

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