Magneto-optical effects in magnetic white dwarfs — I.
The line spectra

Brian Martin Department of Applied Mathematics, Faculty of Science, Australian National University, Box 4, P.O., Canberra, ACT 2600, Australia
D. T. Wickramasinghe Royal Observatory, Blackford Hill, Edinburgh EH9 3HJ, Scotland

Received 1980 October 10; in original form 1980 July 11

Summary. The impact of magnetic birefringence or magneto-optical effects on the line spectra of magnetic white dwarfs is investigated using model calculations. It is shown that at fields at which the line splitting resembles that of an overlapping Zeeman triplet, the central depth of the \( p \) component can be increased by as much as 50 per cent. Changes are also found in polarization near the central component.

1 Introduction

In a recent series of papers, we have attempted to model the spectra of magnetic white dwarfs under various simplifying assumptions (Martin & Wickramasinghe 1978; Wickramasinghe & Martin 1979; see also the confirmatory study of O'Donoghue 1980). We have shown that in most cases, a centred or slightly off-centred dipole field distribution gives results in reasonable agreement with observations of magnetic white dwarfs. However, full detailed agreement is lacking. The ultimate expectation is that with the development of more sophisticated models, it will be possible to determine unambiguously the surface field distribution from an analysis of spectroscopic and spectropolarimetric data. One important simplification in our models has been the neglect of magneto-optical effects in the radiative transfer problem. In this paper we investigate the conditions under which magneto-optical effects can have a significant influence on the line intensity and polarization spectra of magnetic white dwarfs.

The equations describing the transfer of polarized light in the presence of a magnetic field are written normally in terms of the four Stokes parameters \( I, Q, U \) and \( V \) (Hardorp, Shore & Wittmann 1976):

\[
\frac{dI}{d\tau} = \eta_I(I - B) + \eta_Q Q + \eta_V V,
\]

(1)

\[
\frac{dQ}{d\tau} = \eta_Q(I - B) + \eta_I Q - \rho_R U,
\]

(2)
Here $\tau$ is the optical depth, $\mu = \cos \theta$, where $\theta$ is the angle between the propagation direction and the axis along which $\tau$ is measured, $B$ is the local source function and

$$\eta_r = \frac{1}{2} \eta_p \sin^2 \psi + \frac{1}{4} (\eta_l + \eta_r) (1 + \cos^2 \psi),$$

$$\eta_Q = \left[ \frac{1}{2} \eta_p - \frac{1}{4} (\eta_l + \eta_r) \right] \sin^2 \psi,$$

$$\eta_V = \frac{1}{2} (\eta_r - \eta_l) \cos \psi,$$

where $\psi$ is the angle between the propagation direction and the direction of the magnetic field and $\eta_p$, $\eta_l$ and $\eta_r$ are the ratios of the total absorption coefficient of the three shifted Zeeman components plus the shifted continuum absorption coefficient to the (unshifted) continuum absorption coefficient. The solution pair

$$\begin{pmatrix} Q \\ U \end{pmatrix}$$

to equations (1)–(4) must be multiplied by

$$\begin{pmatrix} \cos 2\phi & -\sin 2\phi \\ \sin 2\phi & \cos 2\phi \end{pmatrix},$$

where $\phi$ is the azimuth with respect to an arbitrary $x$-axis at right angles to the line-of-sight.

The effect of the magnetic field is to make $\eta_r$ and $\eta_l$ different from $\eta_p$, hence to make $\eta_Q$ and $\eta_V$ non-zero and so produce non-zero linear polarization of magnitude $(Q^2 + U^2)^{1/2}/I$, circular polarization ($= V/I$) and the normal Zeeman triplet splitting of absorption lines in the intensity $I$.

The terms $\rho_R$ and $\rho_W$ introduce magneto-optical effects. $\rho_R$ leads to a rotation of the electric vector of linearly polarized light, while $\rho_W$ leads to a phase retardation between linear polarizations which are parallel and perpendicular to the magnetic field (Wittmann 1974). Terminology is listed in Table 1.

<table>
<thead>
<tr>
<th>General</th>
<th>Magneto-optical effects</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Magnetic optical birefringence</td>
</tr>
<tr>
<td></td>
<td>Elliptical birefringence</td>
</tr>
<tr>
<td></td>
<td>Anomalous dispersion</td>
</tr>
</tbody>
</table>

$\rho_R$ only (longitudinal magnetic field: $\psi = 0$, therefore $\rho_W = 0$)

- Faraday effect or Faraday rotation
- Induced circular birefringence
- Macaluso–Corbino effect

$\rho_W$ only (transverse magnetic field: $\psi = \pi/2$, therefore $\rho_R = 0$)

- Voigt effect
- Induced linear birefringence
- Cotton–Mouton effect

Table 1. Terms used in referring to magneto-optical effects.
Equations (1)—(4) have been studied by a number of authors both in the absence of magneto-optical effects (Unno 1956) and in their presence (Kai 1968; Beckers 1969; Stenflo 1971; Wittmann 1974, 1977; Hardorp et al. 1976). Most of this work has dealt with either the special, mathematically tractable case of a source function $B$ linear in optical depth $\tau$ or with specific illustrative cases. In this paper we investigate the impact of magneto-optical effects on the emergent line spectrum of realistic models of magnetic white dwarfs. Throughout we use the analytical solution to equations (1)—(4) presented by Martin & Wickramasinghe (1979).

2 Theory

To incorporate magneto-optical effects in lines, it is easiest to start with a Voigt line profile:

$$H(a, v) = \frac{a}{\pi} \int_{-\infty}^{\infty} \frac{\exp \left[ -\left( \frac{v + v^2}{y^2 + a^2} \right) \right]}{y^2 + a^2} dy.$$  
(8)

Here $a = \gamma \lambda^2/4\pi c \Delta \lambda_D$ is the damping parameter, $v = \Delta \lambda / \Delta \lambda_D$ is the position within the profile and $\Delta \lambda_D$ is the Doppler width. $\rho_R$ and $\rho_W$ in the presence of such a component of a line and a magnetic field are given by

$$\rho_R = -\eta_0 (F_r - F_l) \cos \psi,$$
(9)

$$\rho_W = -\eta_0 \left[ F_p - \frac{1}{2} (F_l + F_r) \right] \sin^2 \psi,$$
(10)

where $\eta_0$ is the value of $\eta_p$ (or $\eta_l$ or $\eta_r$) when $a = v = 0$ and

$$F(a, v) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{y \exp \left[ -\left( \frac{v - v^2}{y^2 + a^2} \right) \right]}{y^2 + a^2} dy.$$  
(11)

is the dispersion function, $F_p = F(a, v - v_p)$, $F_l = F(a, v - v_l)$ and $F_r = F(a, v - v_r)$ where $v_p$, $v_l$ and $v_r$ are the amounts by which components $p$, $l$ and $r$ are shifted in the ambient magnetic field. As in the case of $\eta_l$, $\eta_p$ and $\eta_r$, the values of $\rho_R$ and $\rho_W$ in the presence of an absorption line are obtained by summing over the different line components using appropriate strengths and weighting factors and using appropriate shifts $v_p$, $v_l$ and $v_r$ for each component.

$F(a, v)$ clearly is closely related to $H(a, v)$:

$$F(a, v) = a^{-1} \left[ \frac{1}{2} v H(a, v) + \frac{1}{4} \frac{dH(a, v)}{dv} \right].$$  
(12)

Note that $\rho_R$ and $\rho_W$ are analogous to $\eta_Q$ and $\eta_V$, respectively, with $\eta_0 F$ replacing $H$ and a reversal of signs. However, while $H$ is symmetric in $v$, giving rise to the familiar symmetric line profiles, $F(a, v) = -F(a, v)$ and hence $\rho_R$ and $\rho_W$ can give rise to non-symmetric effects.

In the literature there are some differences in nomenclature, as noted in Table 2. In all cases the red component of absorption has $\Delta m = m_{\text{upper}} - m_{\text{lower}} = -1$ and is calculated using $H(a, v - v_l)$ and $F(a, v - v_l)$ where $i = p$, $l$ and $r$. In emission the polarization is such that the $E$ vector at a fixed point in space rotates clockwise as the electromagnetic wave travels in the same direction as the $B$ vector. This is called in classical optics right circularly polarized (RCP) light, as is done by Hardorp et al. (1976), but is called left circularly polarized (LCP) light by various astrophysicists. (We have adopted the later notation,
Table 2. Notations used in referring to red and blue Zeeman components.

<table>
<thead>
<tr>
<th>Author</th>
<th>Component</th>
<th>Δm = +1, blue</th>
<th>Δm = −1, red</th>
<th>Corrections</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unno (1956)</td>
<td>r (right)</td>
<td>l (left)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beckers (1969)</td>
<td>b (blue)</td>
<td>r (red)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stenflo (1971)</td>
<td>b (blue)</td>
<td>r (red)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wittman (1974, 1977)</td>
<td>r (right)</td>
<td>l (left)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hardorp et al. (1976)</td>
<td>l (left)</td>
<td>r (right)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Martin &amp; Wickramasinghe (1978, 1979)</td>
<td>r (right)</td>
<td>l (left)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

consistent with our previous publications.) The blue component of course is associated with opposite signs and rotations.

3 Results

It is straightforward to make the calculations of line absorption with and without ρ_R and ρ_W. However, since understanding the nature of the effect is not so easy, we first use a simple case to illustrate how the effects arise. In Fig. 1 are H and F in a hypothetical isolated

![Figure 1](image-url)

**Figure 1.** Parameter values for a hypothetical Zeeman triplet with \(\lambda (\nu = 0) = 5000 \text{ A} \), T−r relation taken from the zero-field, high-gravity \((\log g = 8.0)\) model atmosphere for a DA white dwarf with \(T_e = 20000 \text{ K}\) given by Wickramasinghe (1972), continuum opacities \(\eta_p = \eta_T = \eta_r = 4\) independent of optical depth, \(\eta_o = 10000\), \(\nu_r = -16\), \(\nu_p = 0\), \(\nu_l = 16\), \(a = 0.1\), \(\mu = 0.8\), \(\cos \phi = 0.7\) and \(\cos 2\phi = 0.6\). Illustrated are: (a) \(\eta_r\), \(\eta_p\), and \(\eta_l\); (b) \(\eta_r\), \(\eta_Q\), and \(\eta_V\); (c) \(F_r\), \(F_p\), and \(F_l\); (d) \(\rho_R\) and \(\rho_W\); (e) intensity \(I\) calculated without \(\rho_R\) and \(\rho_W\) (curve 3) and with \(\rho_R\) and \(\rho_W\) (curve 4); (f) linear polarization without and with \(\rho_R\) and \(\rho_W\) (curves 3 and 4 respectively); (g) circular polarization without and with \(\rho_R\) and \(\rho_W\) (curves 3 and 4 respectively). The normalizations are arbitrary.
Figure 1 — continued
Zeeman triplet with $\eta_I$, $\eta_Q$, $\eta_V$, $\rho_R$, $\rho_W$ and the resultant intensity, linear and circular polarizations illustrated. The solutions without $\rho_R$ and $\rho_W$ can be understood qualitatively on the basis of the Unno solution:

$$I_U = B_0 \left(1 + \frac{\beta \mu \eta_I}{\eta_I^2 - \eta_Q^2 - \eta_V^2}\right),$$  \hspace{1cm} (13)$$

$$Q_U = -B_0 \frac{\beta \mu \eta_Q}{\eta_I^2 - \eta_Q^2 - \eta_V^2},$$  \hspace{1cm} (14)$$

$$V_U = -B_0 \frac{\beta \mu \eta_V}{\eta_I^2 - \eta_Q^2 - \eta_V^2},$$  \hspace{1cm} (15)$$

where $B = B_0 (1 + \beta \tau)$. Although not correct for the non-Unno $T-\tau$ profile used here, equations (13)–(15) give the general character of the solution. The main effect of $\rho_R$ and $\rho_W$ is in the region of the $p$-component (near $v = 0$) of the triplet, where the emitted intensity is lowered, the magnitude of linear polarization is reduced and positive and negative bulges in circular polarization are introduced. These effects can be explained in general mathematical terms as follows.

For the central, $p$-component, $\eta_V = 0$. Hence when $\rho_R = \rho_W = 0$, equations (1)–(4) show that absorption will only affect $I$ and $Q$. When $\rho_R$ and $\rho_W$ are introduced as in Fig. 1(d), $U$ becomes negative through the term $\rho_R Q$, $Q$ is reduced in magnitude (though still remaining negative) through the term $-\rho_R U$, and $I$ is reduced through the term $\eta_Q Q$; other interactions of course contribute to the final result. The net effect is to decrease the magnitude of linear polarization $[(Q^2 + U^2)^{1/2}/I]$ and reduce $I$. On either side of the centre of the $p$-component, the non-symmetric term $\rho_W U$ causes the bulges illustrated in Fig. 1(g).

For the red component, $\eta_V \neq 0$ and $\eta_Q \neq 0$ so that $Q$ and $V$ are non-zero even when $\rho_R = \rho_W = 0$. When $\rho_R$ and $\rho_W$ become non-zero, there is little change in the solution because $U$ remains very small compared with $I$, $Q$ and $V$. To see this, first note that the solution ratio $Q/V$ at any depth approximately equals $\eta_Q/\eta_V$, as is apparent from equations (14) and (15). Second, note that according to equations (6), (7), (9) and (10), if $F \propto H$, then $\rho_R/\eta_V = \rho_W/\eta_Q$. Equation (12) shows that for fixed $v$, $F \propto H$ may not be a bad approximation away from the centre ($v = 0$) of a component giving rise to $H$ and $F$, so that in equation (3), $\rho_R Q - \rho_W V \propto \eta_Q I$ or $\eta_V I$ in the $r$-component, therefore $U \propto Q$ or $V$ and hence $\rho_R$ and $\rho_W$ have little effect on the solution. The same considerations apply to the blue component. Note also from Fig. 1(d), (f) and (g) that the signs of $\rho_R$, $\rho_W$, $Q$ and $V$ are always such that $U \propto Q$ or $V$ in both red and blue components.

We have just described some general mathematical reasons why the $p$-component primarily is affected by $\rho_R$ and $\rho_W$ from adjacent $p$- and $r$-components or from adjacent $p$- and $l$-components. We have also found that there is no noticeable effect due to $\rho_R$ and $\rho_W$ from adjacent $r$- and $l$-components.

The results illustrated in Fig. 1 show an impact on the spectrum of a magnetic white dwarf due to magneto-optical effects as large as any which is likely to occur. Our results indicate that when the field is stronger and the components are more widely separated than in Fig. 1, $\rho_R$ and $\rho_W$ from each component are smaller in the region of the other components and the effect on the spectrum is smaller. But when the field is weaker and the components are closer together, $\rho_R$ and $\rho_W$ as well as $\eta_Q$ and $\eta_V$ are smaller in magnitude compare with $\eta_I$ — see equations (9), (10), (6) and (7) — and this again reduces the influence of $\rho_R$ and $\rho_W$ on the result. For zero field, $F_r = F_l = F_p$ and therefore $\rho_R = \rho_W = 0$.

© Royal Astronomical Society • Provided by the NASA Astrophysics Data System
In Fig. 2 we present an illustrative case of the impact of magneto-optical effects on the spectrum of a magnetic white dwarf star calculated using a realistic model. The model atmosphere is again taken from Wickramasinghe (1972), but in this case results are given for flux and polarizations averaged over the surface of the star assuming a magnetic dipole field distribution, using realistic opacities varying with optical depth, the full set of components for the hydrogen absorption line Hα using shifts $v_l$, $v_t$ and $v_r$ from Kemic (1974), and polarization of the continuum (Lamb & Sutherland 1974).

The results in Fig. 2 should only be used to indicate the likely impact of magneto-optical effects in the absorption lines of magnetic white dwarfs. In particular, we have not attempted here to produce accurate values for polarization in the extreme line wings, both because the Voigt line profile used here does not adequately represent real absorption wings and because magneto-optical effects in the continuum will be important in the far wings. We plan to treat the latter effect in a following paper.

For Hα which resembles an overlapping Zeeman triplet at the fields used to calculate Fig. 2, the qualitative effects are quite similar to those obtained in the simplified model illustrated in Fig. 1. The main effects are a deepening of the central component, a slight reduction in the magnitude of linear polarization and the introduction of antisymmetric
Table 3. Linear and circular polarization across Ha.

<table>
<thead>
<tr>
<th>$\lambda$ (Å)</th>
<th>Linear polarization (without $\rho$)</th>
<th>Circular polarization (without $\rho$)</th>
<th>Linear polarization (with $\rho$)</th>
<th>Circular polarization (with $\rho$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6000</td>
<td>0.000011</td>
<td>-0.0038</td>
<td>0.000054</td>
<td>-0.0038</td>
</tr>
<tr>
<td>6100</td>
<td>0.000054</td>
<td>-0.0057</td>
<td>0.000094</td>
<td>-0.0050</td>
</tr>
<tr>
<td>6200</td>
<td>0.00042</td>
<td>-0.0069</td>
<td>0.00059</td>
<td>-0.0064</td>
</tr>
<tr>
<td>6300</td>
<td>0.00061</td>
<td>-0.0159</td>
<td>0.00131</td>
<td>-0.0164</td>
</tr>
<tr>
<td>6400</td>
<td>-0.0037</td>
<td>-0.051</td>
<td>-0.0042</td>
<td>-0.050</td>
</tr>
<tr>
<td>6500</td>
<td>0.00075</td>
<td>0.00023</td>
<td>0.0026</td>
<td>0.0057</td>
</tr>
<tr>
<td>6600</td>
<td>-0.0058</td>
<td>-0.0028</td>
<td>-0.0038</td>
<td>-0.0066</td>
</tr>
<tr>
<td>6700</td>
<td>0.00133</td>
<td>0.035</td>
<td>-0.00023</td>
<td>0.032</td>
</tr>
<tr>
<td>6800</td>
<td>0.00070</td>
<td>0.0103</td>
<td>0.00025</td>
<td>0.0109</td>
</tr>
<tr>
<td>6900</td>
<td>0.000099</td>
<td>-0.00065</td>
<td>-0.00013</td>
<td>-0.00030</td>
</tr>
<tr>
<td>7000</td>
<td>0.000131</td>
<td>-0.0024</td>
<td>-0.000088</td>
<td>-0.0018</td>
</tr>
<tr>
<td>7100</td>
<td>0.000028</td>
<td>-0.0036</td>
<td>-0.000051</td>
<td>-0.0035</td>
</tr>
<tr>
<td>7200</td>
<td>0.000024</td>
<td>-0.0037</td>
<td>0.000025</td>
<td>-0.0037</td>
</tr>
</tbody>
</table>

bulges in circular polarization near the $p$-component. The changes in polarization caused by the inclusion of magneto-optical effects can be judged by the results presented in Table 3. It is interesting to note that a qualitatively similar behaviour by circular polarization across a triplet occurs even in the absence of magneto-optical effects if one assumes an off-centred dipole field distribution (Wickramasinghe & Martin 1979). It is accordingly important to include magneto-optical effects when determining information on the field structure from polarization data.

We find that when the Zeeman components become widely separated, magneto-optical effects are not important. However, in most high field cases, there is a significant overlapping between Zeeman components from different lines. In these circumstances, the depths of the $p$-components could be significantly increased by magneto-optical effects.

4 Conclusion

We conclude that magneto-optical effects can significantly alter both the polarization and intensity spectra of magnetic white dwarfs. When the line splitting resembles that of an overlapping triplet, we have shown that the central depth of the $p$-component can increase by as much as 50 per cent. Antisymmetric bulges in circular polarization are introduced near the $p$-component, an effect which could be misinterpreted as due to field geometry.

Acknowledgments

DTW acknowledges support from the Australian Research Grants Committee while working in the Department of Applied Mathematics, Faculty of Science, Australian National University.

References


© Royal Astronomical Society • Provided by the NASA Astrophysics Data System
Magneto-optical effects in magnetic white dwarfs – I


