A Mathematical Modelling Course for Advanced Students

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In the second semester of 1980 I gave a course on mathematical modelling to four third and fourth year honours students in applied mathematics. Because of the small size of the class, it was convenient to run this course rather as a research seminar, with the "lectures" mostly comprising presentations made by the students and myself and with much discussion.

For each of the first six lectures, one person posed in advance a simple modelling exercise and everyone brought along solutions or solution methods for discussion. A different person wrote up the discussion for circulation. The problems proposed turned out to be in the areas of resource use, traffic flow, inventory, war, gene breeding and crystal growth. This was one of the best sections of the course. It made everyone aware of the difficulties in designing models for the simplest of problems, and of the irrelevance of many high-powered mathematical techniques for most actual problems. The students were each given the challenge of designing a suitable exercise, and this caused a bit of creative anguish. Because there was no assessment at this stage, there was an opportunity to get used to asking obvious questions and trying out speculative ideas, things which are, or should be, part of any actual modelling process but which are often inhibited in mathematics courses.

The second six lectures were devoted to presenting and analyzing models from the research literature. For each lecture, one person selected and circulated a research paper from the literature beforehand, gave a short presentation about the paper, and later prepared a brief summary of the key points involved. A different person was assigned to lead discussion, focussing on the model's assumptions, realism, adequacy and value. Leading a critical discussion is something with which most students have had little experience. The papers chosen covered muscle-induced ejection flows, complexity and stability of a large eco-system, fold morphology, natural selection among two species with hybrids, and resource allocation in presidential campaigns.

In the third set of six lectures, each person formulated an original problem and developed a model for it, circulated a short account of a solution, and made a short presentation. After the discussion, the person prepared and circulated a revised version of the solution if necessary. Again, thinking up a suitable problem posed real difficulties for nearly everyone. Solutions to these problems were much more developed than those of the first six lectures since roughly two weeks rather than two days of preparation time was available. Topics treated were optimum thickness of glass in a window, strategy for harvesting a paddock, fuel consumption in an accelerating vehicle, trajectory of a cricket ball, and growth of bureaucracies.

The fourth set of six lectures focussed on model assumptions and modelling methodology. For each lecture, one person selected and circulated some material which emphasized model assumptions (in econometrics and in change in voting behaviour) or comparison of methods (in dispersion in flowing media, in fluid flows, and in neutron attenuation) and, as before, gave a short presentation.

In each of the groups of six lectures, there were five papers or exercises treated, by the four students and myself. In my contributions I tried to set a good example, but not an unrealistically difficult one. The sixth lecture in each group was devoted to summary and dissection of the modelling process itself, based on the models and experience we had had by that stage. Occasionally some of my own research papers in the modelling area were used as a basis for analysis; I tried to provide an honest practicioner's view of modelling, describing the aspects of modelling research that are not normally reported in research papers.

The final fourteen lectures were devoted to projects: choosing and studying a particular mathematical model in depth, writing a short paper of about ten pages, and preparing and presenting a short seminar of fifteen to twenty minutes. The students had the option designing their own problem and model, of analyzing and elaborating a model from the research literature, or of studying model assumptions or modelling methodology. As it turned out, the topics shosen, tumour growth, trend surfaces, and cell differentiation, were all based on prior research work. The quality of the work done was excellent, but I probably should have provided more encouragement for the students to develop their own models. The difficulty here is that, in developing a model from scratch, what one can produce in a given amount of time looks much less impressive than a development, variation, or critique of an existing, and often sophisticated, model.

A fair bit of time was devoted to refining both the short paper and the short talk. I read and offered comments on as many versions of the papers that the students cared to prepare, and feedback was offered on dry runs of the talks. This again is something that is seldom done in undergraduate courses. It is vitally important, however, for students to be able to communicate technical information accurately and concisely by writing and by speech wherever they end up working, be it in government, industry, or academia.

Assessment was based entirely on the final paper and seminar, assuming, though not assessing, full participation in the earlier parts of the course. So as to reduce a bit the problems arising from the same person's being both teacher and examiner, I based the assessment largely on comments by outside referees who read the papers. This process was accomplished with surprising speed and goodwill. Using outside referees, though subject to several potential pitfalls, mimics to some extent the actual assessment processes found in scientific work situations.

The general approach of the course probably would be viable for groups of up to eight or ten students. For larger groups than this, it would be advisable to break into smaller groups for most of the time. In any case, it is necessary to trust the students to put in a reasonable amount of thought and work. The students can learn a lot from the exercises and papers that the others pose and present, and at the end of the course everyone has a useful file of modelling exercises, solutions, research papers from the literature, notes on the papers, and the students' own papers.

The role of the teacher is to provide a framework for study which offers stimulating opportunities, not to be the final source of wisdom on every point. Indeed, with so many topics covered, it is impossible to understand every point and to answer every question. A bit of humility can be useful in such a situation!

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