Wind-Load Correlation and Estimates of the Capacity Credit of Wind Power: An Empirical Investigation.

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Abstract

Most previous investigations of the capacity credit of wind power in electricity grid systems have paid little attention to the effect of correlation between wind power and electrical demand. Calculations made with several years of data from Western Australia indicate that empirical estimates of capacity credit may vary greatly with small changes in the joint distribution of wind and load. These changes are not reflected in the ordinary sample correlation coefficient. It is shown that capacity credit estimates are very sensitive to the availability of wind power at a few periods of high load, and a simple summary measure is described which gives an indication of the strength of wind-load association in relation to capacity credit.

1. Introduction

When wind-powered electricity generating plant is introduced into an electricity grid system, the requirement for both conventional fuel and capacity is reduced. The saving in conventional capacity, the so-called 'capacity credit' of wind power, has been studied by a number of authors (Refs. 1-8). At low penetrations of wind power into the grid, the capacity credit approximately equals the average wind power, while at very high penetrations the credit reaches an asymptotic value which depends on the loss of load probability in the absence of wind power and the probability of zero wind power (Refs. 3,8).

Work on capacity credit can be based on several different approaches, including simulations, numerical probabilistic models (Refs. 1-7) and analytical probabilistic models (Ref. 8). Simulations offer the greatest potential for realism but at the expense of computer time and general insights. Analytical probabilistic models can be used to obtain general insights at the expense of realism, while numerical probabilistic models are intermediate in these respects.

An assumption of most of the probabilistic models developed so far is that the wind speed and the electricity load are uncorrelated. Recently, Carlin (Ref. 9) has presented an analytical model for estimating capacity credit in the presence of correlation between wind speed and load, utilising an assumption that the square root of the wind speed and the load can be described by a bivariate normal distribution. The results show, as expected, that the capacity credit increases with increasing correlation, as measured by the usual cross-product correlation coefficient.

This theoretical analysis stimulated us to look for confirmation of the effects of wind-load correlation on capacity credit using 8 years of hourly wind and load data from Western Australia. We found, somewhat surprisingly, that there seems to be little relation between the correlation coefficient (of wind speed – or wind power – and load) and the capacity credit calculated for each year of this data. We also found that the capacity credit estimates vary considerably between different years despite the fact that the correlation coefficient was consistently small (in the range 0.0 to 0.1).

Clearly the assumption of bivariate normality alone provides an insufficient basis for the estimation of capacity credit on a year-by-year basis from this data. In particular we observe that the numerically determined capacity credit is very sensitive to irregularities in the tail of the wind-load distribution. Wind power available at a time of very high load makes a very large contribution to reducing the probability of loss of load. In contrast, the availability of wind at times of low and medium loads makes no difference to the reliability of the grid.

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These considerations led us to the conclusion that in empirical calculations the most useful parameter is not the overall correlation between wind and load, but some measure of the availability of wind power at times of very high load.

In section 2 we describe a simple such measure, with its theoretical rationale, and in section 3 we present some data and numerical calculations. In the final section we outline practical implications of the approach.

2. Theory

Consider an electricity grid with a certain conventional generating capacity C, of which at any given time A is available, that is not undergoing planned or forced outage. If L is the load at a given time, then the fraction of time that the availability A is insufficient to meet the load L is the loss of load probability p_0:

$$p_0 = \Pr(A<L). \tag{1}$$

If wind power of rated capacity W_r and actual power output W at a given time is introduced into the grid, the loss of load probability is reduced to

$$p_W = \Pr(A+W<L). \tag{2}$$

One measure of capacity credit, called Equivalent Firm Capacity, is the hypothetical 'firm' capacity F (that is, capacity with zero outage rate) which satisfies

$$p_W = p_F \equiv \Pr(A+F<L). \tag{3}$$

It may be assumed that A is independent of (and thus uncorrelated with) the other two variables L and W. Thus p_0 can be evaluated by a simple convolution. In general this may be written

$$p_0 = \int_{\ell} \Pr(A<\ell) f_L(\ell)\, d\ell \tag{4}$$

(where $f_L(\ell)$ is the probability density of the load L), while the corresponding numerical computation is

$$p_0 = \frac{1}{N} \sum_{i=1}^{N} \Pr(A<\ell_i) \tag{5}$$

where $\{\ell_i : i=1,\ldots,N\}$ are the recorded values of load (typically a time sequence). The distribution of A is modelled numerically using information on the size and outage rates of generating plant in the grid (Ref. 3). Most of the significant contribution to p_0 will come from the largest few values in $\{\ell_i\}$.

To calculate p_W when W and L are not statistically independent, it is only necessary to replace ℓ_i in (5) by $(\ell-w)_i$, in other words to use the recorded values of load reduced by the concurrent wind power. A similar change applies to (4). Here, however, we wish to obtain a simple expression for p_w that does not require this full calculation and which offers some physical insight into the relation between p_w and p_0.

The following approximation appears to be useful. We assume that whenever the wind power available is nonzero this is sufficient to prevent loss of load. This is exact in the limiting case when W_r becomes very large and is a reasonable approximation when W_r is not too small, since for most loss of load events the value of $L-A$ is much less than the value of A. Then

$$p_W/p_0 \cong \Pr(A<L \text{ and } W=0)/p_0 = \Pr(W=0 \mid A<L), \tag{6}$$

where $\Pr(\cdot \mid A<L)$ denotes conditional probability given $A<L$. In words, when wind power capacity is included, the probability of loss of load can be approximated by the probability that simultaneously $A<L$ and $W=0$. This equals p_0 ($=\Pr(A<L)$) times the probability that $W=0$ given $A<L$.

This is a generalisation of the result obtained in (Ref. 8) since, if W and L are independent, then

$$\Pr(W=0 \mid A<L) = \Pr(W=0),$$

and we recover the result of (Ref. 8), obtained in an analytical probabilistic framework, for the case $W_r \to \infty$:

$$p_W/p_0 = \Pr(W=0) . \tag{7}$$

The conditional probability in (6) is best interpreted as a weighted probability of zero wind power, giving most weight to those times for which the probability of loss of load is highest. This is clearly reflected in the computational form of (6) which may be written

$$p_W/p_0 = \frac{1}{N} \sum_{i=1}^{N} \Pr(W=0 \mid L=\ell_i) \Pr(A<\ell_i)/p_0 , \tag{8}$$

where $\Pr(W=0 \mid L=\ell_i)$ has the value 1 or 0 according to whether some (nonzero) wind power is or is not available at time i.

The right-hand side of (8) we will refer to as the LOLP-weighted probability of zero wind power. This quantity can be expected to provide a reasonable indication (by comparison with the overall probability of zero wind power) of how using the joint wind-load distribution will affect estimates of capacity credit. Its calculation requires parallel wind and load data along with a model of the electricity grid being considered, from which the distribution of A is determined.

3. Observations and Calculations

To test the usefulness of (8), we utilised hourly wind speeds at Fremantle, Western Australia, and corresponding hourly loads (obtained by averaging half-hourly load data) for the state of Western Australia for the years 1971-1978. To make the load data for the different years comparable, each year's data was scaled to a mean $\bar{L}$ = 1000MW. The standard deviation of each year's load was also adjusted to give p_0 = 0.001 for the grid described below.

The grid used was an idealised set of 40 units of 50MW each, with forced outage rates and planned outage rates each equalling 0.10. Hence C = 2000MW and the mean availability $\bar{A}$ = 1600MW. Wind power capacity was introduced, assuming aerogenerators with start-up speed $v_s = 0.53\bar{v}$, rated speed $v_r = 1.50\bar{v}$, linear power response between v_s and v_r, the furling speed $v_f = 25\ ms^{-1}$. Using the empirical distribution of the Fremantle wind speeds, these aerogenerator characteristics gave a capacity factor of 0.48. Besides the limiting case $W_r \to \infty$ used to obtain saturation capacity credits, for more realistic values we chose W_r = 105, 209 and 418MW ($\bar{W}$=50, 100 and 200MW).

Table 1 presents results for the case $W_r \to \infty$. Each year of data was used separately for the various calculations and they were also all combined to give a ninth 'case'. The data sets are ordered by increasing value of the LOLP-weighted probability of zero wind power, given in column 2. The estimates of capacity credit from the joint wind-load distributions (denoted F_J), shown in column 3, were calculated by using the paired values of L and W to construct an empirical distribution of L–W, and hence determine p_W from (2). The estimates assuming independent wind and load distributions (denoted F_I), shown in column 5, were calculated using simple convolution of the empirical marginal distributions of L and W to arrive at p_W (see ref 1). In each case the capacity credit (Equivalent Firm Capacity) F was determined in the following manner. A table of values of p_F was constructed using different values of F in (3). Then by interpolating the value of p_W in the table of p_F–F values, a value of the capacity credit F was determined.

Column 5 in Table 1, giving capacity credit using independent wind and load distributions, shows fairly constant values for the different years, reflecting the relative constancy of the distribution both of wind speed and of load from year to year. But when the calculation using the joint wind-load distribution is made (column 3), the F_J values vary by a factor of well over 2 between 1978 and 1972.

As expected from the theory outlined in section 2 there is a close relationship between the LOLP-weighted probability of zero wind power (column 2) and F_J (column 3), and also between the overall Pr(W=0) (column 4) and F_J (column 5). For example, taking the 1978 data, p_0 = 0.001 by construction, the empirical value of $\Pr(W=0 \mid A<L)$ is 0.071, and hence we expect to find $p_W = 7.1\times10^{-5}$ as $W_r \to \infty$. The value of p_W obtained empirically in order to reach the value F = 145.4MW in

Table 1 was 7.2×10^{-5}, in satisfactory numerical agreement with the theory. The relationship between Pr(W=0) and F_I is not perfectly monotonic only because of small empirical differences in the load distribution from year to year, resulting in differences in the table of $p_F - F$ values in which p_W is interpolated.

The other striking feature of Table 1, apart from the large variations in the capacity credit estimates of column 3, is the fact that these estimates bear little or no relation to the correlation coefficient between wind speed and load, shown in column 6. The latter values are all positive and small, yet the values of F_J vary from almost 50% greater than, down to 40% less than, the values of F_I. This emphasises the enormous sensitivity of the results to small changes in the empirical joint distribution of the data, changes which are certainly not reflected in the ordinary correlation coefficient.

Table 1 For 8 different years of wind and load data from Western Australia, assuming effectively infinite wind power penetration into a grid with mean load 1000MW, listed are the year (column 1), the LOLP-weighted probability of zero wind power (column 2), the capacity credit calculated using the joint wind-load distribution (column 3), the probability of zero wind power (equally weighted over the data) (column 4), the capacity credit calculated assuming independent wind and load distributions (column 5), and the correlation coefficient of wind speed and load (column 6)

Year	Pr(W=0 | L>A)	F_J (MW)	Pr(W=0)	F_I (MW)	correlation coefficient
1978	0.071	145.4	0.189	98.7	0.047
1977	0.100	122.0	0.161	96.8	0.064
1974	0.166	103.7	0.194	91.8	0.030
1973	0.233	86.8	0.203	92.0	0.071
1975	0.351	67.7	0.229	91.0	0.025
1971	0.362	59.4	0.189	92.5	0.038
1976	0.398	57.6	0.253	83.4	0.011
1972	0.410	54.4	0.207	91.0	0.031
all data	0.262	79.5	0.203	91.9	0.040

In Table 2 we report results for realistic wind power contributions to the grid. As expected, away from the limiting case $W_r \to \infty$ the LOLP-weighted probability of zero wind power provides a somewhat less accurate indication of the value of F, due to the failure of the assumption that whenever W>0 this is sufficient to prevent loss of load.

The sensitivity of the capacity credit results to wind speeds at times of high load was confirmed by a simple test. During the two highest load events in 1978, a substantial wind speed was recorded. When the wind speed at these two times was artificially set to zero, and a wind power penetration of 10% assumed ($\overline{W}$=100MW), the capacity credit F_J was reduced from 74.5MW to 49.8MW.

Janssen (Ref. 10) has used Netherlands data for wind speeds and load to calculate a range of measures for system reliability. An "unexpected result" (Ref. 10, p. 47) obtained was that calculated system reliability was slightly improved by using independent wind and load distributions rather than a joint wind-load distribution, in spite of a small positive correlation between the two distributions. This result might well be explained by the effects noted here.

Table 2 For 8 different years of wind and load data from Western Australia, again assuming a mean load of 1000MW, listed are the year (column 1), the capacity credit calculated assuming independent wind and load distributions and $\overline{W}$ = 200MW (column 2), the capacity credit calculated using the joint wind-load distribution and $\overline{W}$ = 200, 100 and 50MW (columns 3, 4 and 5 respectively), and the LOLP-weighted probability of zero wind power (column 6)

Year	F_I (MW) $\overline{W}$=200MW	F_J (MW) $\overline{W}$=200MW	F_J (MW) $\overline{W}$=100MW	F_J (MW) $\overline{W}$=50MW	Pr(W=0| L>A)
1978	73.5	100.0	74.5	45.3	0.071
1977	74.9	93.7	76.3	49.2	0.100
1974	64.8	89.6	71.7	52.1	0.166
1973	74.7	77.6	61.1	41.8	0.233
1975	70.7	63.0	53.5	34.5	0.351
1971	73.4	43.8	36.7	26.9	0.362
1976	65.4	43.3	34.1	26.0	0.398
1972	72.2	41.5	35.2	24.0	0.410
all data	71.2	63.3	52.3	37.0	0.262

4. Conclusions

Our most important conclusion is that empirical estimates of capacity credit may vary dramatically from year to year when proper account is taken of the joint variability of the wind speed and electricity load. Such estimates are particularly sensitive to the availability of wind power at a few periods of high load, when the probability of conventional plant failing to meet demand is greatest. Estimates using a small number of years of data are not sensitive to the overall correlation coefficient between wind power (or wind speed) and load.

It is not clear whether the analytical model of Carlin (Ref. 9), which predicts a direct relation between capacity credit and the correlation coefficient, would prove to be satisfactory as an average over sufficiently long periods of time.

These results indicate that considerable uncertainty may surround a value of capacity credit estimated from a single year's data, especially if the correct calculation has been made, using hourly values of load minus wind power (L–W). On the other hand, estimates made from the marginal distributions of L and W, assuming they are independent, may be quite inaccurate. The index described above, the LOLP-weighted probability of zero wind power, gives an indication of the direction in which these latter capacity credit estimates need to be revised to reflect the effect of using the joint wind-load distribution. Calculation of the index is considerably simpler than the full capacity credit calculation using the values of L–W. Furthermore, data sets with simultaneous wind-load measurements will not always be available.

Further work is needed to clarify how best to obtain meaningful long term estimates of capacity credit from empirical data.

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