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POLARIZATION ANGLE IN MAGNETIC WHITE DWARFS

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ABSTRACT

In magnetic white dwarfs, the angle of linear polarization is observed in some stars to be roughly constant except for a change by 90° at a particular wavelength. One proposed explanation for this phenomenon, the presence of cyclotron absorption, is examined and rejected. Another explanation relying on shifted Balmer jumps is proposed, and applied to Grw +70°8247.

Subject headings: polarization — stars: magnetic — stars: white dwarfs

I. INTRODUCTION

The existence of magnetic fields in white dwarfs can be inferred from several observations. First, the absorption lines may exhibit resolvable Zeeman structure. Second, the continuum and line radiation may be circularly or linearly polarized or both. For low field ($B \lesssim 5 \times 10^7$ gauss) magnetic white dwarfs where the Zeeman theory of the relevant lines and the theory of the continuous absorption coefficients are well established, the wavelengths and strengths of absorption lines and polarization of lines and continuum often can allow a reasonably accurate value of the magnetic field of the star to be inferred, as well as some other parameters such as the orientation of the star's magnetic axis with respect to the viewer (Wickramasinghe and Martin 1979; Angel 1978; O'Donoghue 1980; Landstreet 1979).

However, progress along similar lines has not been possible with the higher field objects due to the absence of suitable Zeeman theory, and various indirect methods for measuring field strength have been proposed. In this paper we investigate the interesting proposal by Angel (1979) that the observed 90° change of the polarization angle in Grw +70°8247 (Landstreet and Angel 1975), and possibly also in GD 229 (Green and Liebert 1981), is due to cyclotron resonance at the wavelength of change. We first describe the physical and mathematical significance of this angle, and then investigate and reject the cyclotron interpretation. We propose instead that the change occurs at the position of a shited Balmer jump. Finally we present a rough analysis of the linear polarization data of Grw +70°8247 using this interpretation.

Radiative transfer in a magnetic field is described by the following equations involving the four Stokes parameters I, Q, U, and V (Hardorp, Shore, and Wittmann 1976):

$$\mu \frac{dI}{d\tau} = \eta_I(I-B) + \eta_Q Q + \eta_V V , \tag{1}$$

$$\mu \frac{dQ}{d\tau} = \eta_Q(I-B) + \eta_I Q - \rho_R U , \tag{2}$$

$$\mu \frac{dU}{d\tau} = \rho_R Q + \eta_I U - \rho_W V , \tag{3}$$

$$\mu \frac{dV}{d\tau} = \eta_V(I-B) + \rho_W U + \eta_I V , \tag{4}$$

where τ is the optical depth, $\mu = \cos\theta$ where θ is the angle between the propagation direction and the axis along which τ is measured, B is the local source function, and

$$\eta_I = \tfrac{1}{2}\eta_p \sin^2\xi + \tfrac{1}{4}(\eta_l + \eta_r)(1 + \cos^2\xi) , \tag{5}$$

$$\eta_Q = [\tfrac{1}{2}\eta_p - \tfrac{1}{4}(\eta_l + \eta_r)] \sin^2\xi , \tag{6}$$

$$\eta_V = \tfrac{1}{2}(\eta_r - \eta_l) \cos\xi , \tag{7}$$

where ξ is the angle between the direction of propagation and the direction of the local magnetic field and η_p, η_l, and η_r are the ratios of the respective sums of the total absorption coefficients of the three shifted Zeeman components and the shifted continuum absorption coefficients to the unshifted continuum absorption coefficient. The quantities ρ_R and ρ_W are the magneto-optical parameters. In Hardorp *et al.*'s (1976) formulation (1)–(4), the solution pair (Q, U), written as a row vector, must be multiplied by

$$\begin{bmatrix} \cos 2\phi & \sin 2\phi \\ -\sin 2\phi & \cos 2\phi \end{bmatrix} . \tag{8}$$

The angle ϕ is the azimuth with respect to an arbitrary x-axis at right angles to the line of sight. The relation of ϕ to the star's geometry is described in the appendix.

When there is no magnetic field, $\eta_r = \eta_l = \eta_p$; therefore $\eta_Q = \eta_V = 0$ and the radiative transfer is described by (1) without the last two terms. A magnetic field leads to values of η_r and η_l different from η_p, and equations (1)–(4) then lead to Zeeman splitting in the intensity I of the absorption lines and circular polarization V/I. The linear polarization, in which we are most interested in this paper, has magnitude $(Q^2 + U^2)^{1/2}/I$, oriented at an angle $\phi = \frac{1}{2}\arctan(U/Q)$ with respect to the polar axis of the star. When determining the flux and polarization values for a star, the Stokes parameters are averaged over the surface, with appropriate weighting factors for area and viewing angle, and the linear polarization angle is

$$\bar{\phi} = \tfrac{1}{2}\arctan(\bar{U}/\bar{Q}) . \tag{9}$$

There may seem to be some circularity in the definition of the linear polarization angle $\bar{\phi}$. As noted before, in the solution to the radiative transfer equations (1)–(4), the solution pair (Q, U) is multiplied by the matrix (8), in which ϕ is the azimuth with respect to an *arbitrary* x-axis perpendicular to the line of sight. In Figure 1 and in our calculation we take this x-axis to be coincident with the meridian of zero longitude: at this longitude the matrix (8) is a diagonal matrix. In calculating the

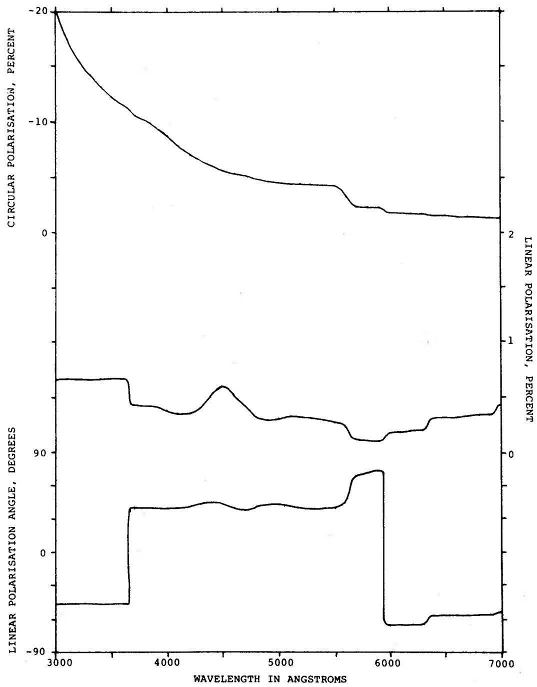

FIG. 1.—Linear polarization angle in degrees (*bottom*), linear polarization in percent (*middle*), and circular polarization in percent (*top*) for a model magnetic white dwarf as described in the text. The viewing angle $i = 15°$.

polarization angle $\bar{\phi}$ from (9), the rotation by the matrix (8) seems to be essentially reversed, giving back the original value of $\bar{\phi}$, which is arbitrary. But this is true only for a single point on the star's surface. In (9) the values $\bar{U}$ and $\bar{Q}$ are the solutions from (1)–(4) averaged over the star's surface, in which at each different longitude a different value of ϕ is used. Because the original x-axis chosen for determining ϕ is arbitrary, the final value $\bar{\phi}$ is also arbitrary in its absolute value. But changes in $\bar{\phi}$ are still significant, since changes in the solutions Q and U are significant once the arbitrary x-axis is assigned.

The physical significance of $\bar{\phi}$ is that it measures the orientation of the plane of linear polarization in the sky. Determining the reason for changes in $\bar{\phi}$ by understanding its physical origin is not easy because of the complexity of the physical origins of polarized light. We prefer to investigate this by looking instead at the mathematical formulation.

II. CYCLOTRON ABSORPTION?

Can cyclotron absorption explain the 90° change in polarization angle, as claimed by Angel (1979)? Here we give a generall but we think conclusive argument that it cannot.

To investigate the effect of cyclotron absorption on $\bar{\phi}$, it would be desirable to do calculations with opacities and magneto-optical parameters near the cyclotron frequency. The problem is essentially to determine a suitable expression for the dielectric permittivity tensor in a magnetic field, and use this to calculate the opacities and magneto-optical parameters (Shafranov 1967). Classical expressions for the tensor are readily available (Ginzburg 1961; Shafranov 1967; Akhiezer *et al.* 1975), though they are quite complicated near the cyclotron frequency. A classical treatment incorporating collisions adds much extra complexity (Pacholczyk 1976; Pavlov, Mitrofanov, and Shibanov 1980). But to treat the problem properly, a quantum-mechanical treatment is needed. Lamb and Sutherland (1974) give a quantum-mechanically derived expression for the cyclotron opacity alone. A quantum-mechanical expression for the dielectric permittivity tensor has been given by Pavlov, Shibanov, and Yakovlev (1980). But this latter expression would be extremely difficult to use to calculate opacities and magneto-optical parameters, and in addition it applies only to high-temperature stars and away from the core of the cyclotron resonance.

It seems that a detailed treatment of this problem would be a major project. Fortunately our main conclusion follows from much simpler considerations.

First consider the cyclotron absorption cross section (Lamb and Sutherland 1974):

$$\sigma_+ = 4\pi^{3/2} \frac{e^2}{mc} \frac{1}{\omega} \left[\frac{mc^2}{(2kT \cos^2 \xi)} \right]^{1/2} \times \left[1 - \exp\left(\frac{-h\omega}{2\pi kT} \right) \right]^{-1} \exp\left[\frac{-mc^2(\omega - \omega_G)^2}{(2kT\omega^2 \cos^2 \xi)} \right], \quad (10)$$

in which ω is the frequency and $\omega_G = eB/mc$ is the cyclotron frequency. This cross section applies only to right-hand circularly polarized light; in other words, it contributes only to η_r. The essential characteristics of σ_+ are that it is very strong and very narrow. The narrowness arises from the final exponential term. We have shown previously (Martin and Wickramasinghe 1979) that, in the absence of magneto-optical effects, this form of σ_+ can in most circumstances have only a minor effect on the spectrum of a magnetic white dwarf. At a given wavelength, σ_+ will absorb significantly only for a narrow range of magnetic fields; and if the field distribution of the star departs from uniformity, as in the case of a dipole, then significant absorption will occur only for a tiny fraction of the star's surface.

Lacking a quantum-mechanical calculation of the magneto-optical parameters near a cyclotron resonance, consider the following classically derived expressions (Pacholczyk 1976):

$$\rho_R = -\omega_0{}^2 \omega_G \cos \xi / [ck_c(\omega^2 - \omega_G{}^2)] , \quad (11)$$

$$\rho_W = -\omega_0{}^2 \omega_G{}^2 \sin^2 \xi / [2c\omega k_c(\omega^2 - \omega_G{}^2)] , \quad (12)$$

where $\omega_0 = (4\pi Ne^2/m)^{1/2}$ is the plasma frequency and k_c is the unshifted continuum absorption coefficient per unit volume. Clearly these expressions break down near the cyclotron frequency ω_G, but they do suggest that ρ_R and ρ_V greatly increase near ω_G.

Equations (10)–(12) tell us that near the cyclotron frequency the opacity is dominated by η_r and that ρ_R and ρ_W are also very large. The next step is to determine what effect this has on the polarization angle.

To study this, we are primarily concerned with the solutions Q and U in equations (2) and (3). To obtain a qualitative idea of the behavior of these solutions with respect to values of opacities and magneto-optical parameters, it is sufficient to look at the Unno (1956) solution to the radiative transfer equations, which is based on the assumption of a source function B which is linear in optical depth τ. Let us look at the ratio U/Q for the Unno solution (see Martin and Wickramasinghe 1982):

$$U/Q = (\rho_W \eta_V - \rho_R \eta_Q)\eta_I / [\eta_Q(\eta_I{}^2 + \rho_W{}^2) + \rho_R \rho_W \eta_V] \, . \quad (13)$$

Except in the very narrow region near the cyclotron resonance, ρ_R and ρ_W will be much larger than the opacity parameters η_I, η_Q, and η_V, which are each of order η_r due to σ_+ (see [5]–[7]). Hence away from ω_G, $U/Q \sim 0$. If, very near ω, η_r is much larger than ρ_R and ρ_W, the same result holds. Only when η_r is comparable with ρ_R and ρ_W will U/Q be much different from 0. But this can at most be in a narrow region where σ_+ is significant, which is a negligible fraction of the surface of the star. The conclusion from this is that the value of linear polarization is dominated by the value Q resulting from (2). The value U from (3) will almost always be negligible by comparison.

The averaging process over the star's surface, and other parameters relevant to the final value of the linear polarization, do not affect this conclusion.

i) *Latitude.*—As noted before, for a given wavelength only a narrow range of magnetic fields, and therefore a narrow range of latitudes with respect to the magnetic axis, will potentially give anything other than $U/Q \sim 0$. Averaging over latitudes is the process which washes out the effect of both cyclotron absorption and rotation of the linear polarization angle.

ii) *Longitude.*—At each different longitude with respect to the magnetic axis, the matrix (8) by which the result (Q, U) is multiplied is different. It is the net result of summing results from different longitudes (and latitudes) that gives the values $\bar{Q}$ and $\bar{U}$ for insertion into (9). The values ϕ for use in the rotation matrix (8) are independent of wavelength, opacities, and other parameters, and thus cannot give rise to a change in the result $U/Q \sim 0$.

iii) *The angle* ξ.—The angle ξ between the direction of propagation and the direction of the magnetic field affects the opacity parameters via (5)–(7), but this does not affect the general result $U/Q \sim 0$.

iv) *Crossing the cyclotron resonance.*—The cyclotron opacity σ_+ given by formula (10) does not change sign across the cyclotron frequency. The magneto-optical parameters in (11) and (12) both change sign across the cyclotron frequency, and this causes the sign of U to switch but not the signs of I, Q, or V (see Martin and Wickramasinghe 1982, eqs. [15]–[19]). The result $U/Q \sim 0$ therefore holds on both sides of the resonance.

Let us now put together the effects noted so far. For almost all parts of the star, $U/Q \sim 0$, and hence from (8) the value $\bar{Q}$ is the appropriately weighted average of $Q \cos 2\phi$ across the surface of the star and the value $\bar{U}$ is the appropriately weighted average of $Q \sin \phi$. Going from one side of the resonance to the other, the values of Q do not change sign and the averaging process over latitude and longitude remains constant. As a result the polarization angle $\bar{\phi}$ cannot change either. Hence cyclotron absorption cannot explain the 90° change in polarization angle.

III. BALMER EDGES

The above analysis superficially suggests that there is no mechanism for changing the polarization angle, since $\bar{Q}$ and $\bar{U}$ are averages, weighted in an identical manner across the surface of the star, of $Q \cos 2\phi$ and $Q \sin 2\phi$, and the ratio of these averages will always be the same sign. The key to this apparent dilemma is the arctan function in (9): if both the numerator and denominator in (9) change sign, the arctan is taken from a different quadrant 180° away, and the polarization angle $\bar{\phi}$ changes by precisely 90°. Hence all that is required to obtain the required change in polarization angle is for the result Q in (2) to change sign. Cyclotron absorption does not cause such a change and hence is ruled out.

It is possible to obtain a change in the sign of Q by using the classical expressions for the dielectric permittivity tensor. For example, Ginzburg's (1961) expressions give a 90° change in polarization angle, in the special case of transverse propagation ($\xi = \pi/2$), for $\omega^2 = \omega_G{}^2/3$ (Landstreet 1983). However, this result has little relevance to the stars presently under discussion. The derivation includes only free-free transitions with collisions in the presence of an external magnetic field (hence including cyclotron absorption), and applies for frequencies much greater than the collision frequency and the plasma frequency. It assumes a uniform temperature and generates only a small value of the opacity in comparison to atomic opacity sources which are dominant in these stars. Most importantly, the quantum-mechanical expression for cyclotron absorption and the averaging over the star's surface—which together extinguish the significance of cyclotron absorption for polarization angle—are not included.

Derivations utilizing classical expressions can provide some insight into the qualitative possibilities for polarization effects. This is the approach adopted by Gnedin and Sunyaev (1974), who show in general terms how the linear polarization angle can change by 90° due to variations in continuum opacities. However, they mainly treat this as a theoretical possibility.

We follow Gnedin and Sunyaev in looking to variations in continuum opacities to explain shifts in polarization angle at high magnetic fields. For magnetic white dwarfs in the optical spectrum, we believe the most plausible candidate for causing the actually observed 90° changes in polarization angle is a shifted Balmer edge. Assuming a linear Zeeman theory, the values of the continuum opacity are given by (Lamb and Sutherland 1974):

$$\sigma_q[\omega] = \omega(\omega - q\omega_L)^{-1}\sigma_0[\omega - q\omega_L] \,, \tag{14}$$

where $\omega_L = \omega_G/2$; $q = 0, +1$, and -1 for η_p, η_r, and η_l, respectively; and the square brackets enclose the angular frequency at which σ is evaluated. For $\omega \gg \omega_L$, the opacity parameter η_p is unchanged, η_l at frequency ω is the value η_p at $\omega + \omega_L$, and η_r at frequency ω is the value η_p at $\omega - \omega_L$. Consider a field sufficient to cause the Balmer edge for η_l to shift to about 5500 Å. Then blueward of 5500 Å η_l will be the largest continuum opacity, and redward of 5500 Å it will be the smallest. From equation (6) and (7), this will cause η_Q and η_V to change signs across 5500 Å, and hence Q will change sign also. This provides the basis for a change in sign of $\bar{Q}$ and a change in $\bar{\phi}$ by 90°. But if the field strength is nonuniform, this change will not occur precisely or smoothly at 5500 Å, since the wavelength of the Balmer jump depends on the magnetic field strength and hence on latitude. But the jump will still be fairly sharp, since as long as one sign of Q dominates over the surface of the star and $U/Q \sim 0$, $\bar{\phi}$ will remain constant.

To illustrate this effect, we have carried out model calculations of linear polarization, some results of which are shown in Figures 1, 2, and 3. A magnetic white dwarf with centered dipole field is assumed, the polar strength taken as $H_d = 4 \times 10^8$ gauss. The atmospheric structure is the zero-field, high-gravity ($\log g = 8.0$), $T_e = 15{,}000$ K model from Wickramasinghe (1972). Continuum polarization is calculated according to Lamb and Sutherland (1974). To avoid shifting to negative frequencies, Lamb and Sutherland's formula (14) for $q = +1$ only is replaced rather arbitrarily by

$$\sigma_q[\omega] = \exp(\omega_L/\omega)\sigma_0[\omega \exp(-\omega_L/\omega)] \,. \tag{15}$$

Examining the first two terms in a Taylor series for the exponential functions, it is clear that for $\omega \gg \omega_L$ this is equivalent to (14) with $q = +1$. Magneto-optical parameters are calculated according to Pacholczyk (1976)—see equations (11) and (12). There are no lines. The wavelength at which the polarization angle changes depends somewhat on the viewing angle. A

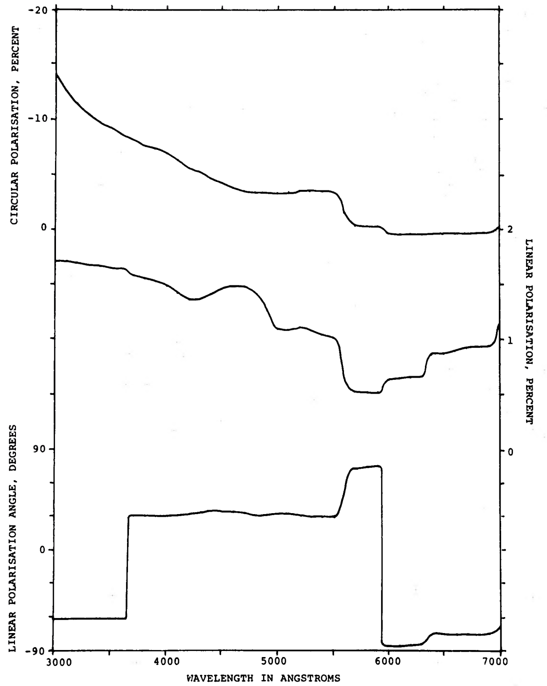

FIG. 2

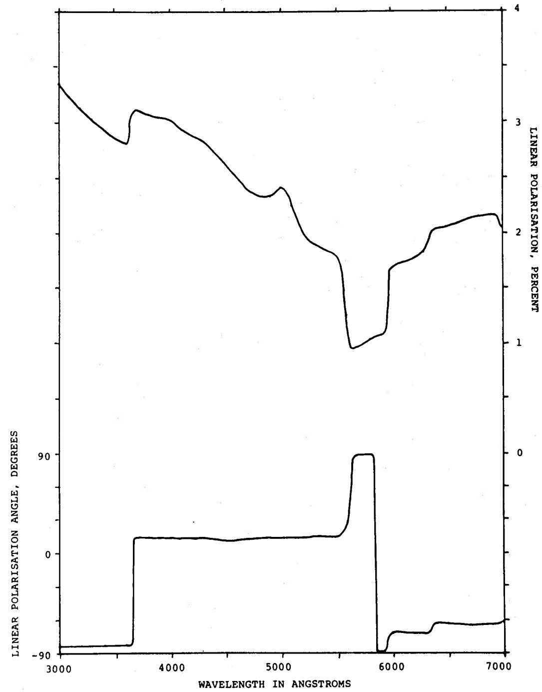

FIG. 3

FIG. 2.—As Fig. 1, with viewing angle $i = 45°$

FIG. 3.—As Fig. 1, with viewing angle $i = 90°$. The circular polarization, not presented, is identically zero.

pole-on viewing angle $i = 0°$ is not used since linear polarization is then identically zero. When viewing from near the direction of the magnetic axis ($i = 15°$), the regions with highest field near the pole are more heavily weighted, and so the wavelength at which the polarization angle changes is somewhat higher than when viewing from the direction of the equator ($i = 90°$). For all viewing angles the polarization angle changes at the $\lambda = 3646$ Å Balmer jump, since with (14) and (15) we have assumed a Zeeman effect in which the p-component of opacity does not shift with magnetic field.

The results in the figures provide an indication of how the optical observations of Grw $+70°8247$ (Landstreet and Angel 1975) may be explained. The observations show the linear polarization angle changing by about 90° at $\lambda \sim 5500$ Å where the linear polarization reaches a minimum value of a few tenths of a percent. The circular polarization decreases with increasing wavelength in the wavelength region 4000–7000 Å. These general characteristics are reproduced by our model with $H_d = 4 \times 10^8$, $i = 15°$. However, the theory used for the continuous opacity is strictly valid only at $B \lesssim 10^8$ gauss, so that no confidence can be placed on the particular value of H_d used to obtain agreement with the wavelength at which the polarization angle is observed to change by about 90° in Grw $+90°8247$. We also note that our results show a change in polarization angle by 90° also at 3646 Å, which is not observed. Again it is likely that the discrepancy is due to our assumption that the p component of opacity remains unshifted at 3646 Å even at the high fields of 2×10^8 to 4×10^8 gauss relevant to this star.

IV. CONCLUSIONS

We have investigated the suggestion by Angel (1979) that the 90° change in the polarization angle observed in Grw $+70°8247$ may be due to cyclotron resonance at the wavelength of change, and have shown that it is incorrect. We have argued that the change occurs at the wavelength of a shifted Balmer edge and have presented some calculations in support of this hypothesis. Bound-free opacity calculations appropriate to the high field regime ($B > 10^8$ gauss) are required before the testing of this hypothesis in relation to Grw $+70°8247$ can be carried out more rigorously.

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APPENDIX

We describe here how to calculate the various angles needed in mathematically representing radiative transfer in a magnetic field. A general point on the star's surface in rectagular coordinates, and normalized to unity, is

$$\boldsymbol{x} = (x, y, z) = (\sin\theta' \cos\phi', \sin\theta' \sin\phi', \cos\theta') ,$$

where θ' is the polar latitude and ϕ' is the longitude measured counterclockwise from the x-axis. The magnetic field $\boldsymbol{B} = (B_x, B_y, B_z)$ is in the case of a dipole field coincident with the z-axis, and normalized to unity,

$$\boldsymbol{B} = (\tfrac{3}{2}\sin\theta' \cos\theta' \cos\phi', \tfrac{3}{2}\sin\theta' \cos\theta' \sin\phi', \tfrac{3}{2}\cos^2\theta' - \tfrac{1}{2})/|B| ,$$

where $|B| = (\frac{1}{4} + \frac{3}{4}\cos\theta')^{1/2}$.

The x-axis has been chosen to be in the plane of the dipole axis (z-axis) and the viewing direction, so that the viewing direction is given by

$$\boldsymbol{k} = (\sin i, 0, \cos i) .$$

The angle θ between the propagation direction and the axis along which τ is measured is given by

$$\mu = \cos \theta = \boldsymbol{x} \cdot \boldsymbol{k} = \sin i \sin \theta' \cos \phi' + \cos i \cos \theta' .$$

The angle ξ between the direction of propagation and the direction of the local magnetic field is given by

$$\cos \xi = \boldsymbol{B} \cdot \boldsymbol{k} = B_x \sin i + B_z \cos i .$$

The angle ϕ is the azimuth with respect to an arbitrary axis at right angles to the line of sight. To determine this azimuth, consider the vector $\boldsymbol{a} = \boldsymbol{k} \times \boldsymbol{B}$, which is always perpendicular to the line of sight. The zero azimuth $\boldsymbol{a}_0$ is taken for $\boldsymbol{B}$ evaluated at $\theta' = i + \pi/2$ and $\phi' = 0$, giving $\boldsymbol{a}_0 = -\boldsymbol{y}$. Then

$$\cos \phi = \boldsymbol{a} \cdot \boldsymbol{a}_0 = (B_z \sin i - B_x \cos i)/|\sin \xi| .$$

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