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A TEST OF THE DIPOLE MODEL FOR THE ROTATING MAGNETIC WHITE DWARF FEIGE 7

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ABSTRACT

Liebert *et al.* have presented observations of flux and circular polarization for Feige 7. They interpreted the star as a hot, rotating magnetic white dwarf. Without using a mathematical model of the star, they utilized a variety of evidence to infer values of the magnetic field strength, field geometry, and rotation angle, among other parameters. In this paper we use a detailed mathematical model of Feige 7 to test Liebert *et al.*'s conclusions. We generally find their inferences to be sound, with several important qualifications and alterations. In particular, we find it impossible to model the observed periodic variation in the positions of the absorption lines using a dipole field.

Subject headings: stars: individual — stars: magnetic — stars: rotation — stars: white dwarfs

I. INTRODUCTION

Since the early 1970s a number of white dwarfs have been reported which show features indicating a significant magnetic field (Angel 1978). The most revealing features are shifts in absorption lines (Zeeman splitting) and significant linear and circular polarization. A number of these stars have been modeled and their features interpreted as due to a magnetic field in the range 10^6–10^8 G, usually in the form of a dipole (sometimes displaced from the center of the star).

Liebert *et al.* (1977) first reported the star Feige 7 (=L795-7=GR 267). The star shows displaced absorption lines, indicating a substantial magnetic field, and also significant circular polarization. Feige 7 is particularly interesting in two ways. First, unlike most magnetic white dwarfs, which show only a hydrogen spectrum, Feige 7 shows Zeeman components of both hydrogen and neutral helium. Second, both the spectrum and the circular polarization vary periodically.

Liebert *et al.* utilized their data to the utmost to infer the relevant parameters about Feige 7. They used the tables by Kemic (1974), which give the shifts in the wavelength of components of absorption lines as a function of magnetic field, to infer a magnetic field and to identify the absorption lines they observed. They estimated the mean longitudinal field strength using an approximate calculation involving the value of the circular polarization. And they inferred the magnetic field geometry from the sinusoidal variation of the circular polarization. Finally, they used their data on the star to make comments on the evolution of white dwarfs.

Leibert *et al.* did not make use of a computer model of Feige 7 in making their inferences. Rather, they used all available information and careful logic, without a detailed model. Our intention here is to test the accuracy of Liebert *et al.*'s analysis in the light of results using a computer model.

Previously, we have developed a computer model for calculating theoretical spectra for magnetic white dwarfs (Martin and Wickramasinghe 1978, 1981, 1982, 1984; Wickramasinghe and Martin 1979*a*). The model relies on an atmospheric structure, which is taken from a standard source. (Thus far we have assumed that the atmospheric structure is not significantly affected by the magnetic field.) The equations of radiative transfer for polarized light are solved to obtain the emergent spectrum (Martin and Wickramasinghe 1979*a*). The continuum opacity is shifted in the magnetic field using the formulation of Lamb and Sutherland (1974). The line opacities for hydrogen and helium are shifted using the data provided by Kemic (1974). The line profiles are taken as Voigt profiles, so that magneto-optical effects can be readily incorporated both in the continuum and the lines (Martin and Wickramasinghe 1981, 1982). The field structure is taken as a combination of multipoles, usually a dipole, which in addition may be offset (Martin and Wickramasinghe 1984). Intensity and polarization values are obtained at a mesh of points across the surface of the star and appropriately weighted and averaged to obtain the resultant flux and polarization.

For the purposes of this paper, the computer model may be taken as a black box. The inputs are:

a) atmospheric structure (e.g., hydrogen, effective temperature 20,000 K);

b) magnetic field strength;

c) magnetic field geometry (e.g., dipolar);

d) viewing angle (angle between the line of sight and the magnetic field axis).

There are in addition a number of inputs specific to the modeling process itself, such as the number of integration points in the optical depth and across the surface of the star.

The outputs of the model are flux, linear polarization, and circular polarization as a function of wavelength. The basic procedure in the modeling process is to make educated guesses about the input parameters—educated by examination of data about the star being modeled—and compare the output with observational data. This procedure is basically the inefficient one of trial and error, since the processes being modeled are nonlinear and dependent on a range of parameters.

We have used our computer model to produce theoretical spectra for Feige 7. In doing so we have tested a number of the arguments and conclusions of Liebert *et al.* (1977). In the rest of this paper we spell out our findings. In the next section we comment on the field strength and structure of Feige 7 and look closely at how absorption lines are formed. In § III we examine circular polarization and the geometry of rotation of Feige 7. The problem of the movement of the positions of the absorption lines is addressed in § IV. In the conclusion we

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1986ApJ...301..177M

point out both the strengths and shortcomings of Liebert *et al.*'s analysis and summarize the inferred characteristics of Feige 7.

II. FIELD STRENGTH AND STRUCTURE

Using our computer model, we have found that most of the features of the data for Feige 7 can be reproduced well with an atmospheric structure with effective temperature 20,000 K, composed of helium and hydrogen in the ratio 100 to 1, with a dipole field of polar surface strength 3.5×10^7 G. The one key feature of Feige 7 that cannot be explained with this model is the magnitude of the periodic variation in the positions of the absorption lines. In § IV we discuss this problem. We will discuss the different features of the model in turn, with detailed comments only when there are significant differences from Liebert *et al.*'s (1977) treatment.

Our model for the atmospheric structure is the $T_e = 20{,}000$ K, log $g = 8.0$ model taken from Wickramasinghe (1972) with He/H = 100. This gives a good representation of the slope of the optical spectrum and the relative strengths of the hydrogen and helium lines (Wickramasinghe and Martin 1979*b*).

The absorption lines in our model are represented by Voigt profiles, shifted according to the tables of Kemic (1974). Previously (Wickramasinghe and Martin 1979*a*) we had difficulty obtaining the correct depths of the absorption lines. In our present model this problem was overcome through two changes. First, our incorporation of magneto-optical effects has led to somewhat deeper lines. Second, we have reduced the Stark widths of the lines to 0.01 of the zero field value to account for the removal of the l degeneracy at high magnetic fields. The reduction factor adopted is arbitrary. It has the consequence that the widths of the computed lines are almost entirely due to magnetic broadening (see Martin and Wickramasinghe 1984).

Liebert *et al.* (1977) concluded that "a large part of the surface must have field strength within ~10% of 18 megagauss" (p. 458). Rather than begin with such an assumption, we tried to model the spectrum with a dipole field before trying more uniform fields. Using dipole fields with different strengths, we found that a dipole strength of 35 MG (with an error of perhaps 1 MG) gave the best fit (see Fig. 1). A centered dipole of strength 35 MG gives a variable field strength (and direction) across the surface of the star ranging from a polar strength of 35 MG to an equatorial strength of 17.5 MG. Thus the equatorial regions of our model roughly correspond to Liebert *et al.*'s 18 MG.

Liebert *et al.*'s conclusion that most of the field is in the range 16–20 MG is based on the observation of the narrowness of the absorption features and the considerable variation in absorbing wavelength of most components as a function of magnetic field. In Figure 2 we present the variation in absorbing wavelength, based on Kemic (1974), for selected helium lines. It would seem that, for example, the most blueward of the components shown would lead to absorption from 4150 Å at 35 MG to 4450 Å at 17.5 MG. How could such a line result in a narrow feature such as shown in Figure 1? Liebert *et al.* concluded correctly that it could not and therefore that the field must be quite uniform. We conclude also that it could not. But we find that a dipole is quite compatible with the observations and that the observed lines could be due to components that are essentially stationary through much of the field variation from 17.5 to 35 MG.

Liebert *et al.* (1977) looked for absorption lines which were absorbing at the appropriate wavelengths at roughly 18 MG. We, by contrast, find that the spectrum can be produced by absorption components which are essentially stationary through much of the field variation. In Figure 2, for example, only those components which are essentially vertical in the field range 17.5–35 MG will contribute to observed features. The most blueward component, mentioned before, will be spread to such an extent that no observable feature will result. Indeed, most components will be spread in this way, the result being a broad, continuum-like depression in the spectrum.

Listed in Table 1 are the components which we identify as generating particular absorption features in Feige 7 assuming a dipole field. In fact, most of these components are the same ones as identified by Liebert *et al.*, since components which are stationary near 18 MG are also those stationary at higher fields. There are several important differences between our identifications and those of Liebert *et al.*

a) The major component causing the 4400 Å feature is an Hγ component rather than the helium component mentioned by Liebert *et al.*

b) For the 4770 Å feature, the helium component mentioned by Liebert *et al.* does not contribute significantly. The

TABLE 1

IDENTIFICATION OF SOME ABSORPTION LINES IN FEIGE 7 ASSUMING A DIPOLE FIELD

Wavelength (Å)	Identification	Comment
4400............	Hγ $\sigma^+ 2p0$, $5g$–1	Dominant contribution
	He $\lambda 4471$ $\pi(0, 0)$, $(1, 1)$, $(-1, -1)$	Lesser contribution
4520............	He $\lambda 4471$ $\sigma^+(-1, -2)$	
4560............	He $\lambda 4471$ $\sigma^+(1, 0)$, $(0, -1)$	Too strong, probably due to extrapolation
4770............	Hβ $\pi 2p1$, $4d1$	
	Hβ $\pi 2p$–1, $4d$–1	
	Hβ $\pi 2s0$, $4p0$	Lower fields only
	Hβ $\pi 2p0$, $4d0$	Higher fields only
4810............	Hβ $\pi 2p0$, $4d0$	Lower fields only
	Hβ $\pi 2s0$, $4f0$	Higher fields only
4920............	Hβ $\sigma^+ 2p$–1, $4d$–2	
4960............	Hβ $\sigma^+ 2p0$, $4d$–1	
	Hβ $\sigma^+ 2p1$, $4d0$	
	Hβ $\sigma^+ 2s0$, $4f$–1	
5030............	He $\lambda 4921$ $\sigma^+(0, -1)$, $(1, 0)$	Not strong enough, probably due to extrapolation
5190............	He $\lambda 5015$ σ^+	Not strong enough, probably due to extrapolation

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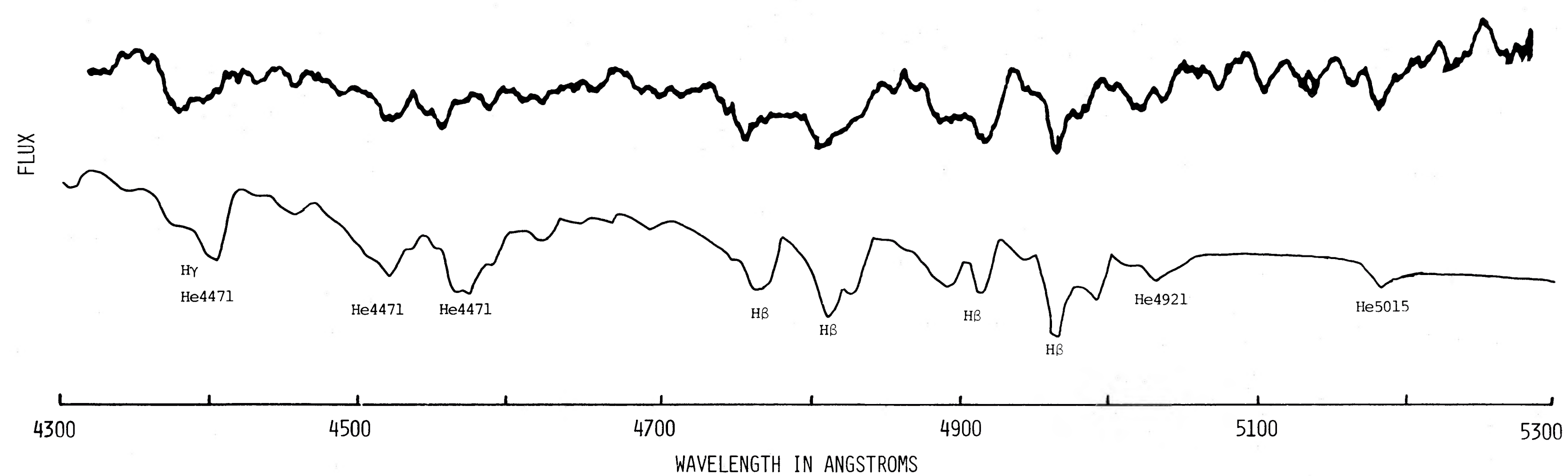

FIG. 1.—(*top*) Observations of flux from Feige 7 at phase 0 from Liebert *et al.*, and (*bottom*) flux as calculated with a centered dipole model with dipole (polar) field strength 35 MG and viewing angle 90°.

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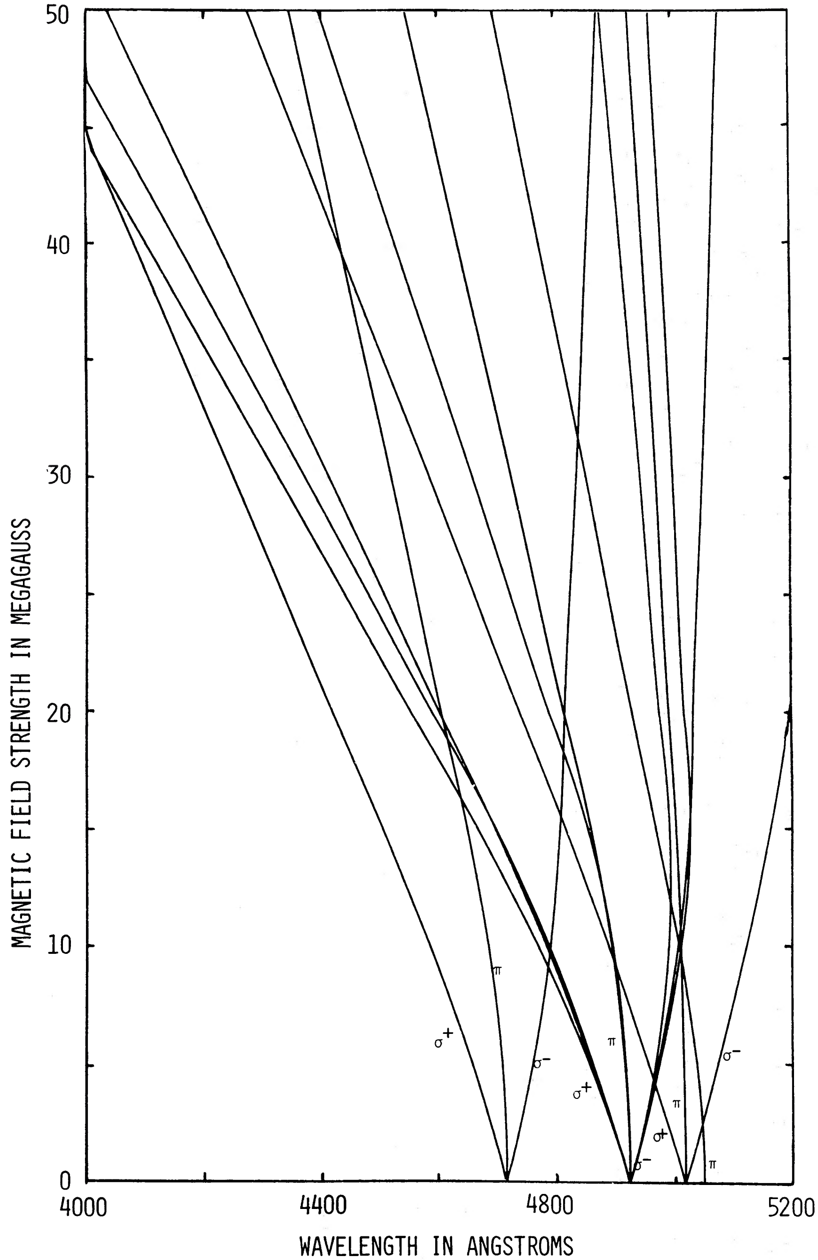

FIG. 2.—Positions of the helium absorption lines He λλ4713, 4921, 5015, and 5048 as a function of magnetic field strength. Some weaker components are omitted. The values are quadratically interpolated from Kemic (1974) up to 20 MG and linearly extrapolated at higher fields.

Hβ $\pi 2s0$, $4p0$ component only contributes at lower fields (near 18 MG), while Hβ $\pi 2p0$, $4d0$, not cited by Liebert *et al.*, contributes at higher fields (up to 35 MG).

c) For the 4810 Å feature, the helium lines mentioned by Liebert *et al.* do not contribute significantly. The Hβ $\pi 2p0$, $4d0$ only contributes at lower fields, while Hβ $\pi 2s0$, $4f0$, not cited by Liebert *et al.*, contributes at higher fields.

d) For the 4960 Å feature, Hβ $\sigma^+ 2p1$, $4d0$ and $2s0$, $4f$–1, not mentioned by Liebert *et al.*, are important contributors.

Our major point is not so much the actual identification of particular components for causing absorption features, as the general conclusion that a dipole field can indeed cause sharp features, but only when the absorbing components are essentially stationary over a range of magnetic fields.

One other point is important. The tables of Kemic (1974) do not give wavelengths for high fields, especially for helium lines. For the values given by Kemic we have interpolated in the field—as shown in Figure 2—using a quadratic fit in the field strength. Beyond the limits of Kemic's tables—which means beyond 20 MG for most helium lines—we have used linear extrapolation. In some cases this gives rise to sharper features than observed, in other cases to weaker features. Our assumption is that extended tables would rectify these inadequacies in the results. For example, in Figure 2 the most redward component is extrapolated in a redward direction, leading to a weak line. But if the component is reality begins to move blueward above 20 MG, as seems quite plausible, then the component would be much stronger, since the absorption would be at similar wavelengths at a wider range of fields. Cases such as this are noted in Table 1.

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1986ApJ...301..177M

III. CIRCULAR POLARIZATION

Liebert *et al.* (1977) observed a variation in Fiege 7's circular polarization which is very close to sinusoidal. The oscillation is around zero, and with the same period as the variations in the wavelengths of the absorption lines. They used the maximum value of the magnitude of circular polarization (0.34%) in an approximate method for determining circular polarization from wavelength, effective temperature, field strength, and viewing angle (Landstreet and Angel 1975) in order to obtain an estimate of the viewing angle. As we have shown (Martin and Wickramasinghe 1979*b*), the method of Landstreet and Angel in general may be inaccurate by a factor of 2, but in the continuum where linear polarization is much less than circular polarization it can be quite accurate. Using our computer model we obtained broad-band circular polarization ($\lambda\lambda$3500–7500) for viewing angles ranging from 0° to 90° (see Table 2), confirming the possibility of a sinusoidal variation. (The viewing angle is the angle between the star's magnetic axis and the line of sight.) By interpolating, we find that according to these figures the viewing angle varies between approximately 60° and 120°. This is in very close agreement with the conclusion of Liebert *et al.*, who give a variation between 66° and 114°.

Liebert *et al.* conclude from the sinusoidal variation of circular polarization around zero, with viewing angle varying from 66° to 114° (i.e., 24° either side of 90°), that the angle between the magnetic and spin axes must be 24°, with the spin axis perpendicular to the line of sight. This is indeed one interpretation (see Fig. 3*a*). But we note that the observations are compatible—and indeed indistinguishable—from another configuration, in which the magnetic and spin axes are perpendicular and the angle between the line of sight and the spin axis is 24° (Fig. 3*b*). While observationally these two configurations are indistinguishable, there are two reasons to believe that the second configuration is more likely.

1. The second configuration does not depend on the spin axis and the line of sight being perpendicular, which is unlikely *a priori.*
2. From the point of view of dynamical stability, the magnetic axis may tend preferentially to be aligned either along or perpendicular to the spin axis (Borra, Landstreet, and Mestel 1982).

Given that the variation in circular polarization can be explained by the noncoincidence of the spin and magnetic axes, does this also explain the variations in the wavelengths of the absorption lines? First consider a centered dipole. The field strength is twice as great at the poles as at the equator. Therefore when the viewing angle is 90° (equator toward the observer) one would expect to see wavelength shifts more characteristic of lower fields (17.5–20 MG), while at viewing angles different from 90° one would expect to see wavelengths shifts characteristic of higher fields. Liebert *et al.* (1977) conclude that this effect cannot be responsible for the variations in wavelengths of the absorption lines. They give two reasons. First, the period of variation of the absorption lines is equal to the period of circular polarization oscillations. If viewing the star's field distribution at different angles were responsible for the wavelength shifts, the variation would be at half the period of the circular polarization oscillation. Second, they argue that the sharpness of the absorption lines is incompatible with a viewing angle greatly different from 90°, since viewing the full range of fields would blur the features. From this it follows (though not spelled out explicitly by Liebert *et al.*) that viewing

TABLE 2

CIRCULAR POLARIZATION FOR THE COMPUTER MODEL OF FEIGE 7[a]

OFFSET FRACTION	CIRCULAR POLARIZATION $i = 60°$	$i = 90°$	$i = 120°$
0...........	−0.0042	0	0.0042
−0.10	−0.0045	−0.00039	0.0040
−0.20	−0.0050	−0.00077	0.0040

[a] Without absorption lines, averaged over 3500–7500 Å. The results including absorption lines are similar.

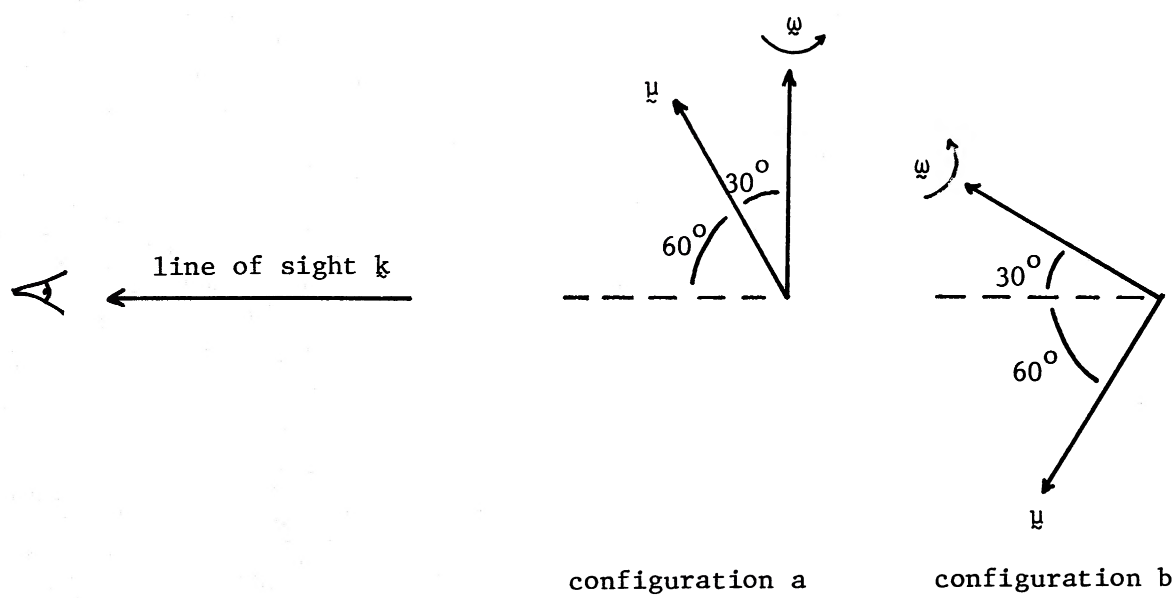

FIG. 3.—Two configurations of the line of sight ($\underset{\sim}{k}$), magnetic axis ($\underset{\sim}{\mu}$), and spin axis ($\underset{\sim}{\omega}$) for a magnetic white dwarf, which are compatible with observations of Feige 7. The configurations are observationally indistinguishable. They are shown at the instant in which $\underset{\sim}{\mu}$ is in the plane of $\underset{\sim}{k}$ and $\underset{\sim}{\omega}$.

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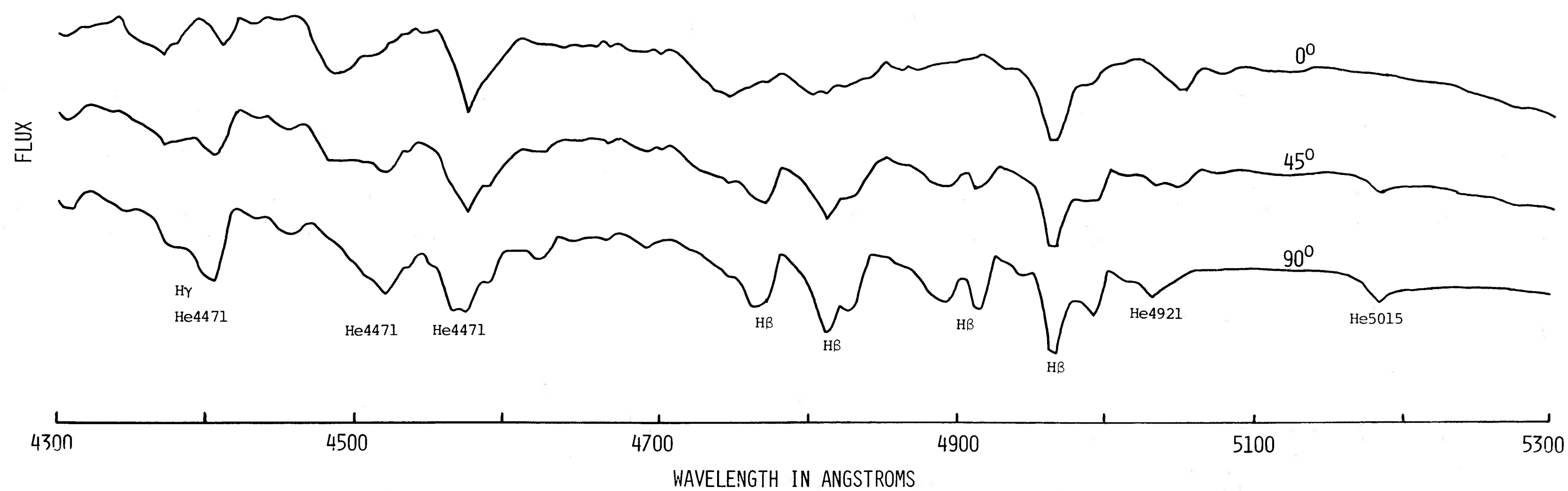

FIG. 4.—Flux as calculated with a centered dipole model with dipole field strength 35 MG, for the viewing angles (*top*) 0°, (*middle*) 45°, and (*bottom*) 90°

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a centered dipole at significantly different angles cannot be the basis for the variation in the wavelength of absorption lines.

We agree with Liebert *et al.* that the variation in absorption lines cannot be explained by rotation of a centered dipole. We find their first reason above—that the periods of absorption line and circular polarization oscillation are equal—to be decisive. But, while agreeing with their conclusion, the second reason above bears examination. There is no need to rule out viewing angles substantially different from 90°, because at every viewing angle there is a contribution to the flux from all different fields on the surface of the star (for a centered dipole). All that changes when the viewing angle changes is that the weighting of the contributions from different parts of the star changes. Thus, even when the viewing angle varies over a considerable range, for many line components the wavelength of absorption is hardly affected, as shown in Figure 4.

IV. FIELD STRUCTURE REVISITED

How then are the variations in the wavelengths of the absorption lines to be explained? Liebert *et al.* (1977) suggest an offset dipole, or alternatively a quadrupole component in a predominantly dipole field. They suggest that offsetting toward the South Pole by 0.016 R_* is sufficient. We find that offsetting a dipole by *any* amount is inadequate to explain the observations.

The shifts in the positions of the absorption lines presented by Liebert *et al.* are large: up to 15 Å in some cases. The question is how to reproduce these shifts. We find using our model that a dipole offset by as much as 0.15 R_*—10 times the offsetting proposed by Liebert *et al.*—produces shifts of only a few angstroms. (This is illustrated in Fig. 5 with a look at the lines near 4800 Å.) The reason for this is simple: the relevant absorption lines are mostly stationary at different field strengths. Offsetting the dipole does change the field strengths quite significantly, but this does not translate into large enough movements of the lines.

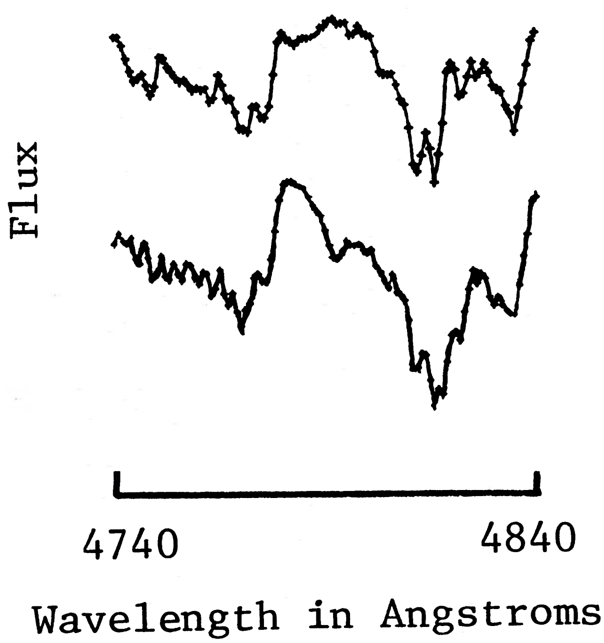

FIG. 5.—Flux as calculated with a dipole model with dipole field strength 35 MG offset 0.15 R_* toward the South Pole, for viewing angles (*bottom*) 60° and (*top*) 120°.

If a dipole were offset even further, say 0.30 R_*, then suitable shifts might be generated. But well before this, the results for circular polarization would become incompatible with observations. Liebert *et al.* carefully analyzed the deviations from a sine curve allowed by the circular polarization data and found them to be small. Our results bear this out: model dipoles offset by 0.15 R_* generate circular polarization results which diverge significantly from the observations.

Replacing the dipole by a quadrupole or higher multipole will not help matters, for the same reasons. Another possibility canvassed by Liebert *et al.*, surface inhomogeneities in the H and He abundances—and found by them not to be likely on Feige 7, also would not generate the shifts in absorption lines, again for the same reasons.

The only resolution that we can envisage is some sort of uniform field whose strength varies almost discontinuously across the surface. For example, if one hemisphere were 18 MG and the other 20 MG, this might be compatible with the observations. On the other hand, a smoothly varying uniform field, such as a field composed of 80% uniform field and 20% *centered* dipole, would not explain the results, since the variation in field strength would not be abrupt enough to produce the absorption line shifts. What would be required is something like 80% uniform field and 20% *offset* dipole, with the offsetting probably being 0.50 R_* or more. We have not generated models with such arbitrary field distributions, since the exercise is much too speculative in the absence of any physical basis for such configurations. Our basic point here is that any smoothly varying field we can imagine, such as multipoles or offset dipoles, cannot simultaneously explain the absorption line shifts and the sinusoidal variation in circular polarization.

Liebert *et al.* concluded that the field must be quite uniform, but they thought this was compatible with a dipole field, which it is not. They also thought that a slightly offset dipole field could produce the absorption line oscillations, which we find cannot be done. Our conclusion here is more negative than positive: a dipole field, or indeed any field varying smoothly across the surface of the star, is very unlikely to be the basis for an explanation of both the sinusoidal variation of circular polarization and the large periodic variation in the positions of the absorption lines. Confirmation of the wavelength shifts reported by Liebert *et al.* is accordingly crucial and would pose a serious problem for the dipole field geometry usually assumed for magnetic white dwarfs.

V. CONCLUSIONS

We have used a computed model to reproduce theoretically the observational data for the magnetic white dwarf star Feige 7. The computer model uses as input an atmospheric structure and magnetic field strength and direction. It incorporates line and continuum absorption as affected by a magnetic field, including magneto-optical effects, and integrates the radiative transfer equations for polarized light both in optical depth and across the surface of the star.

The model parameters which best reproduce the features of Feige 7 include a standard zero-field, line-blanketed atmosphere with the ratio of the helium to hydrogen abundance equaling 100, with $T_e = 20{,}000$ K, log $g = 8.0$, a dipole magnetic field with dipole strength 35 MG. The angle between the magnetic axis and the line of sight can be either 30° or 90°, and the angle between the spin axis and the line of sight is respectively either 90° or 30°.

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We have used our model to test the inferences of Liebert *et al.* (1977), who did not have the benefit of a computer model. Many of Liebert *et al.*'s conclusions are affirmed by the computer model:

a) their conclusions about composition and temperature;

b) most of their identifications of the line components causing particular absorption features;

c) their finding of the minimum field strength;

d) their inference of the range of angles between the direction of the magnetic field and the line of sight.

There are also a number of points about which our model leads to insights different from those of Liebert *et al.*

a) The absorption features can be due to components which are stationary over field strengths ranging from 17.5 to 35 MG rather than just the region 18–20 MG emphasized by Liebert *et al.* Therefore some of our identifications of the components responsible for particular absorption lines are different from those of Liebert *et al.*

b) We identified two possible relations between field axis, spin axis, and line of sight which are compatible with the observations. Only one of these was mentioned by Liebert *et al.* We suggest that the relation not mentioned by them, in which the magnetic and spin axes are perpendicular, is more likely.

c) Liebert *et al.*'s explanation of the way in which variations in magnetic field strength across the surface of the dipole will affect the spectrum needs modification. A small angle between the magnetic axis and the line of sight will not result in so large a shift in absorption features as thought by Liebert *et al.* Similarly, offsetting of a dipole as postulated by Liebert *et al.* cannot reproduce the observed absorption line shifts.

It is clear that careful inferences made about magnetic white dwarfs stars without a computer model—such as done by Liebert *et al.* (1977)—can be quite accurate. By taking into account the insights made possible by use of a realistic computer model, inferences made without a model can be even more accurate.

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