Mathematics and Social Interests

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Mathematics is a product of society and it can both reflect and serve the interests of particular groups. The connection between mathematics and interest groups can be examined by looking at the social construction of mathematical knowledge and by looking at the social system in which mathematics is created and used.

Scientists have long believed that scientific knowledge is knowledge about objective reality. They commonly distinguish their enterprise from religious or political belief systems, seeing scientific truth as unbiased. This belief system has always had difficulties with certain applications of science such as nuclear weapons. The usual way in which the belief in the purity of science is maintained is by distinguishing between scientific knowledge and its applications. Scientific knowledge is held to be pure while its applications can be for good or evil. This is known as the use–abuse model.

This standard picture came under attack in the late 1960s and early 1970s. Radical critics argued that science is inevitably shaped by its social context. For example, funding of pesticide research by the chemical industry arguably influences not only what research topics are treated as important but also what types of ecological models are considered relevant for understanding agricultural systems. Many critics argued that the key motive behind science is profit and social control (Rose & Rose 1976a,b Arditti *et al.* 1980).

The political critics of science drew on and stimulated dramatic changes in the study of the history, philosophy and sociology of science. Thomas Kuhn (1970) opened the door with his concept of paradigms, which are essentially frameworks of standard ideas and practices within which most scientific research proceeds. When a paradigm is overthrown in the course of a scientific revolution, the criteria for developing and assessing scientific knowledge change. The implication is that there is no overarching rational method to decide what is valid knowledge: scientific knowledge depends, on some level, on the vagaries of history and culture.

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Sociologists studying scientific knowledge have developed and filled out this picture. They have examined not only the large-scale political and economic influences on scientific development but also the micro-processes by which scientists 'negotiate' what is scientific knowledge (Barnes 1974, 1977, 1982; Bloor 1976; Latour & Woolgar 1979; Mulkay 1979; Knorr *et al.* 1980).

Most of this analysis has been communicated using social science jargon in specialist journals and has had relatively little impact on practising scientists. The only philosopher of science taken note of by many scientists is Karl Popper, and even his ideas are used more as a 'resource' in struggles over knowledge than as methodological aids (Mulkay & Gilbert 1981). Nowhere is this more true than in mathematics.

What does it mean to talk about the relationship between mathematics and social interests? It can refer to the impact of social factors — such as sources of funding, possible applications or prevalent beliefs in society — on the content and form of mathematical knowledge, such as on the choice of areas to study, the formulation of methods of proof and the choice of axioms. Alternatively, it can refer to the role mathematics plays in applications, from actuarial work to industrial engineering. Finally, it can refer to the social organisation of the production of mathematics: the training of mathematicians, patterns of communication and authority in mathematical work, professionalisation, specialisation and power relations.

'Interest' here refers to the stake of an individual or social group in particular types of actions or social arrangements. An interest can be small-scale, such as the personal advantage to a mathematician in publishing a paper to gain tenure, or large-scale, such as the strategic advantage to a military force in using an algorithm for tracking missiles. 'Social interests' are those associated with major social groupings such as social classes, large organisations, occupational or ethnic groups.

My aim here is to survey some ideas bearing on mathematics and social interests. I approach the problem from two directions. The first is via the sociology of knowledge. Can sociological examination be applied to the creation and elaboration of mathematical knowledge? What does it mean to talk of the social shaping of mathematics? There are some provocative studies in this area, but in my view they do not lead by themselves to a comprehensive picture which can be used to evaluate the role of mathematical work in contemporary society.

The second path involves looking at the system of production and application of mathematical knowledge, and in particular at the use of expertise in modern society and at the relationship between mathematical theory and application.

Path One: Sociology of Knowledge

The sociology of knowledge attempts to explain the origin and evolution of knowledge using the same sorts of analysis which are applied to other phenomena, both natural and social. The dynamics of knowledge involve

social, economic, political, religious, biological and all sorts of other factors. Rather than assuming that the content and structure of knowledge is 'given' by logic or the nature of reality — a transcendental explanation of knowledge — the sociology of knowledge looks for more mundane explanations.

David Bloor (1976) is a leading proponent of the 'strong program in the sociology of science', which aims to investigate all knowledge using sociological methods. The key features of the strong program according to Bloor are that knowledge be explained in causal terms, that explanations be impartial and symmetrical with respect to the truth or falsity of the beliefs being explained, and that the theory be applied to itself.

Bloor adopts an approach to mathematics based on improving John Stuart Mill's view that all mathematics is ultimately based on physical models and human experiences, such as the manipulation of pebbles which can be seen as a motivation for arithmetic with natural numbers (Bloor 1976, Ch. 5). The traditional obstacle to Mill's view is F. L. G. Frege's point that mathematics seems to be 'objective': mathematical reasoning has a compulsion about it which cannot always be attributed to a link with physical models. To extend Mill's theory, Bloor observes that Frege's definition of objectivity is equivalent to social convention: mathematicians have institutionalised a set of beliefs about the ways to proceed with the symbols they work with. These institutionalised beliefs are rather like rules in a game: they *must* be adhered to. Bloor's extension of Mill's perspective is that physical situations provide models for certain steps in mathematical reasoning (usually the more basic features) while mathematical convention gives an obligatory aspect to these steps and extensions of them. Mathematics thus deals not with physical reality but with social creations and conventions.

Bloor's reconstruction of Mill's position provides a powerful basis for the sociological investigation of mathematics. Since the 'law-like' features of mathematical reasoning are based on social conventions, then it is natural to investigate how these conventions are created, sustained and overturned.

Bloor investigates the history of mathematics to see what happened to alternative conceptions of mathematics, dealing with issues such as whether one is a number, Diophantine equations, and Pythagorean and Platonic numbers (Bloor 1976, Ch. 6). His conclusion is that alternative conceptions did exist, but that historians have relegated them to the historical rubbish bin of 'non-mathematics'. In this way only 'genuine mathematics' remains part of the history of mathematics, which thus seems to be cumulative and without significant deviations or alternatives.

Bloor also examines the ways in which mathematical reasoning is socially 'negotiated', namely the practices through which mathematicians develop agreed-upon ways of using and interpreting the symbols and tools of their trade, including criticism, argumentation, reclassification and consensus (Bloor 1976, Ch. 7). Bloor gives among other examples the case of the negotiation, over the years, of the proof of the formula $E + 2 = V + F$ relating the number of edges, vertices and faces of a polygonal solid.

Bloor's program is a powerful one. It opens the foundations of mathematics to sociological examination by allowing the 'objectivity' of mathematical reasoning to be seen as fundamentally social in nature. But Bloor does not extend his analysis to address the relation between mathematics and social interests. Even if it is accepted that the formula $E + 2 = V + F$ depends on somewhat arbitrary agreements among mathematicians rather than being inherent in the nature of polygonal solids (or the mathematical concept of polygonal solids), that does not provide much insight into whether the social negotiation of the formula owes much or provides special benefits to particular groups in society.

At this stage it is worth while to spell out the different channels through which the form and content of mathematics can be shaped by society. Social interests can be connected with the choice of areas of mathematics to study, the interpretation of mathematics and the development of mathematical frameworks.

The choice of mathematical areas to study Differential funding or the availability of applications can affect the opening of branches of study and the prestige of different subjects. For example, the field of operations research grew out of military applications of mathematics during World War II and the strength of the field is maintained by continuing military interest.

Luke Hodgkin (1976) argues that the great surge in the 'mathematics of computation', which encompasses numerical analysis and parts of computer science, is connected to the development of the needs of contemporary capitalism plus the availability of suitable technology for computing (such as transistors and now chips). He points out that the mathematics of computation is not a simple 'reflection' of the economic system, as a simplistic Marxist account might suggest. Instead, the influence of the system of economic production is mediated through the social institutions of science, whose organisation predated the great growth of computational mathematics.

Choice in mathematical research is also involved at the detailed level of application. Partial differential equations can be applied to many problems; the particular sets of equations which are selected out for formulation and solution can be influenced by applications, which in turn are linked to social interests.

The interpretation of mathematics In many cases, especially in applied mathematics, mathematical constructions are chosen because they have desirable physical or social interpretations.

An example here is Paul Forman's (1971) study of the effect of Weimar culture on the development of quantum theory. The most important strides in quantum theory occurred in Germany in the decade after World War I. Forman documents the intense antagonism to rationality which prevailed then in the Weimar Republic. Since causality was identified with rationality, physicists came under pressure to renounce their traditional allegiance to causality. Forman suggests that this pressure led the quantum physicists to search for, or at least latch on to, a mathematical formalism which could be interpreted as non-causal. In crude terms, the acausal Copenhagen interpretation and its associated mathematical framework were adopted because they looked good publicly.

Forman's study is quite relevant to mathematics, since theoretical physics constitutes the foremost application of mathematics. The case of quantum theory is intriguing because, in the decades since the establishment of the orthodox or Copenhagen interpretation, a number of alternative interpretations have been put forth. Some of these use the same mathematical formulations, but interpret their physical significance differently, while others use different mathematical formulations to achieve the same or different results.

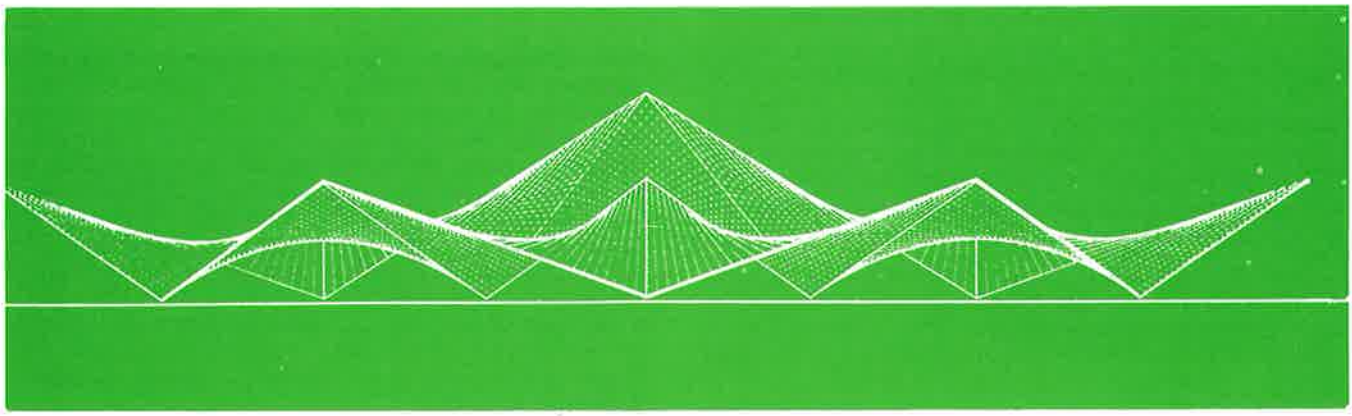
- The statistical interpretation favoured by Einstein uses the same mathematics (Ballentine 1970).
- The hidden variable interpretation, a determinist approach, formulates the equations somewhat differently and, optionally, can give different results from the orthodox theory by addition of an extra parameter (Bohm 1952).

- The splitting universe interpretation is a different interpretation of the same mathematics (DeWitt 1970).
- The 'realist' interpretation, which gets rid of the indeterminist element in quantum theory entirely, uses a different mathematical approach to achieve some of the same basic results (Landé 1965).

The existence of these interpretations or reformulations of quantum theory add support to Forman's analysis. At the least, the interpretation of the equations of quantum theory as supporting indeterminism was not *required* by the equations themselves. Furthermore, it seems possible that many of the achievements of the theory might have been accomplished using a somewhat different mathematical formulation, which could well have been *difficult* to interpret indeterministically.

So strong was the commitment to indeterminism that physicists accepted without question John von Neumann's proof in the 1930s that no hidden variable theory could be constructed. Although Bohm demonstrated such a theory in 1952, it was not until the 1960s that the flaw in von Neumann's proof was exposed (Pinch 1977).

In my experience, most physicists do not worry greatly about what quantum theory 'means' but simply use the mathematics in a pragmatic fashion. Indeed, one of the 'crisis points' commonly experienced by physics students is when they give up their increasingly uncomfortable attempts to understand what the theory *really* means and instead just accept it, usually by sweeping their doubts under the carpet. Most historians and textbook writers have accommodated this process, as Bloor has argued about mathematics history, by exorcising alternative interpretations as unsuccessful, irrelevant or nonexistent.



The development of mathematical frameworks
The choice of axioms, the types of theorems, the style of proofs and a host of other facets of mathematics can be shaped by factors such as views about the nature of social reality.

An example here is game theory, a mathematical theory which deals with conflict situations, originally developed to model economic systems (Martin 1978). Key concepts of the theory include the 'players' in a game, each of which has a number of 'choices', followed by 'payoffs'. The mathematical theory of games is built around determining the optimal strategies for making choices. The players, choices and payoffs are usually assumed to be fixed; competition is built in; payoffs tend to be quantifiable. Hence, game theory is especially suited for applications which assume and reinforce individualism and competition.

Game theory has been applied in many areas, such as international relations. What often happens in practice is that the values of the modellers are incorporated into the game theoretic formulation, which usually ensures that the game gives results which legitimate those very same values. Game theory in this situation provides a 'mystifying filter': values are built into an ostensibly value-free mathematical framework, which thus provides 'scientific' justification for the decision desired. Arguably, game theory has become popular because its mathematical framework makes it easy to use in this way.

The above-mentioned studies and others (Thomas 1972; Ogura 1974; Bos & Mehrtens 1977; MacKenzie 1978; Mehrtens 1987; for a comprehensive survey and analysis see Restivo 1983) show how the social context, such as economics or belief systems, can influence the areas of mathematics which are opened up and made fashionable, the types of theories which are developed, and the particular mathematical formalisms which are formulated and used. These are examples of the impact of social factors on mathematical knowledge, but they hardly establish that all of mathematics is influenced in these sorts of ways. To establish this would require many studies in the line of Bloor's strong program, in an attempt to whittle down the areas of apparent autonomy of mathematical knowledge. Only if the range of sociological studies were very broad could the burden of proof be put on those who claim that there are areas of mathematics which are free of such formative influences.

Even if the strong program could be so developed, what would it say about mathematics and social interests? The existence of influences on the creation and adoption of mathematical knowledge does not automatically mean that knowledge preferentially serves particular groups in society.

The studies in the sociology of knowledge *initiate* the case that mathematics is connected with social interests, by refuting the view that mathematical knowledge always springs antiseptically from the nature of logic, from physical reality or from mathematicians' heads. The limits of sociological examination of mathematics remain to be tested. Some such as Bloor (1981) think the prospects are good while others disagree (Laudan 1981). In any case, since most of the sociology of knowledge studies deal with influences on the origin and development of mathematical knowledge in earlier eras, they only partially address concerns about the uses of present-day mathematics. To pursue the case further, I turn to the second path.

Path Two: The Mathematics–Society System

This approach to looking at mathematics enters not at the level of mathematical knowledge but at the level of the social systems in which that knowledge is created and applied. The social system of science refers to patterns of employment, funding, communication, training, authority, decision-making and so forth. The aim here is to look at the way systems of production and application of mathematics relate to social interests. To do this I select out some salient features of the social systems associated with mathematical expertise.

Sources of patronage Most of the money for mathematics research — which is largely for salaries, but also for offices, libraries, computing and travel — comes from governments and large corporations. The source of funding inevitably has an influence on the areas of mathematics studied and the types of mathematical applications undertaken. As argued by Hodgkin (1976), much of the stimulus for work in computational mathematics also comes from actual or potential military applications.

At the detailed level of application, the formulation of mathematical problems is strongly influenced by funding and opportunities for application. In manufacturing industry, mathematical problems grow out of the need to cut costs, improve technologies or control labour. A mathematical model for the rapid cooling of a metal bar without cracking is tied to an immediate problem. The mathematics of light transmission in optical fibres is driven by interest in application in telecommunications. The number of examples is endless.

What happens in many cases is that a practical problem, such as modelling air pollution dispersion or the trajectories of missiles, leads to a more esoteric mathematical project in numerical analysis or differential equations. The applications, and thus the funding, in these cases have an indirect influence on the type of mathematical problems studied and thought to be 'interesting'. That particular types of parabolic partial differential equations become whole fields of study in themselves is not due simply to some abstract mathematical significance of these equations, but to their significance in practical applications, even if at several stages removed.

Professionalisation Today, most mathematicians — taking a mathematician to be a person who creates or applies mathematical knowledge at a high level — are full-time professionals, working for universities, corporations or governments. There are few amateurs, nor do many mathematicians work for trade unions, as farmers, in churches, or as freelancers. Mathematics, like the rest of science, has been professionalised and bureaucratised. The social organisation of mathematics influences the ways that ambitious mathematicians can pursue fame and fortune (Collins & Restivo 1983).

Mathematicians have a vested interest in their salaries, their conditions of work, their occupational status and their self-image as professionals. Their preferences for types and styles of mathematics are influenced by these factors.

Judith Grabiner (1974) argues that there have been 'revolutions in thought which changed mathematicians' views about the nature of mathematical truth, and about what could or should be proved'. Grabiner examines one particular revolution, the switch from the 1700s when the main aim of mathematicians was to obtain results to the 1800s when mathematical rigour became very important. Of the various reasons for this which Grabiner canvasses, one is worth noting here. Only since the beginning of the 1800s have the majority of mathematicians made their living by teaching. Rather than just obtaining mathematical results for applications or to impress patrons, teachers need to provide a systematic basis for the subject, to aid students but also to establish a suitable basis for demarcating the profession and excluding self-taught competitors from jobs. This is an example of how the social organisation of the profession of mathematics can affect views about the nature of mathematical truth.

Gert Schubring (1981) has argued that in the professionalisation of mathematics in Prussia in the early 1800s, the 'meta-conception' of pure mathematics played an important role. By defining 'mathematics' as separate from externally defined objectives, the mathematicians oriented the discipline to internal values which they could control. To do this, support from the state had to be available first. Given state patronage for academic positions, the mathematicians could proceed to establish a discipline by establishing training which channelled students into the new professional orientation, reducing the number of self-taught mathematicians obtaining jobs in the field, and socialising students into the meta-conception of pure mathematics. This account meshes nicely with that of Grabiner.

This process continues today. Especially in universities, the home grounds of pure mathematics, mathematicians stake their claims to autonomy and resources on their exclusive rights, as experts, to judge research in mathematics. This is no different from the claims of many other disciplines and professions (Larson 1977). The point is that if mathematicians emphasised application as their primary value, their claims to status and social resources would be dependent on the value of the applications and on the evaluations of other professionals in the areas of application. The conception of 'pure' mathematics enables an exclusive claim to control over the discipline to be made.

Herbert Mehrtens (1987, p. 160) develops the thesis and 'a scientific discipline exchanges its knowledge products plus political loyalty in return for material resources plus social legitimacy.' He shows how German mathematicians in the 1930s were able to accommodate the imperatives of the Nazis, especially by providing useful tools to the state. The adaptability of the German mathematics community grew out of its social differentiation, specifically the different functions of teaching, pure research and applied research. Mehrtens' study provides an excellent model for analysing the interactive dynamics of the two factors of patronage and the structure of the profession.

Male domination Most mathematicians are men, and mathematics like the rest of natural science is seen as masculine: a subject for those who are rational, emotionally detached, instrumental and competitive. Mathematicians are commonly thought, especially by themselves, to have an innate aptitude for mathematics, and claims continue to be made that males are biologically more capable of mathematical thought than females. The teaching of pure mathematics as concepts and techniques separated from human concerns, plus the male-dominated atmosphere of most mathematics research groups, make a career in mathematics less attractive for those more oriented to immediate human concerns, especially women.

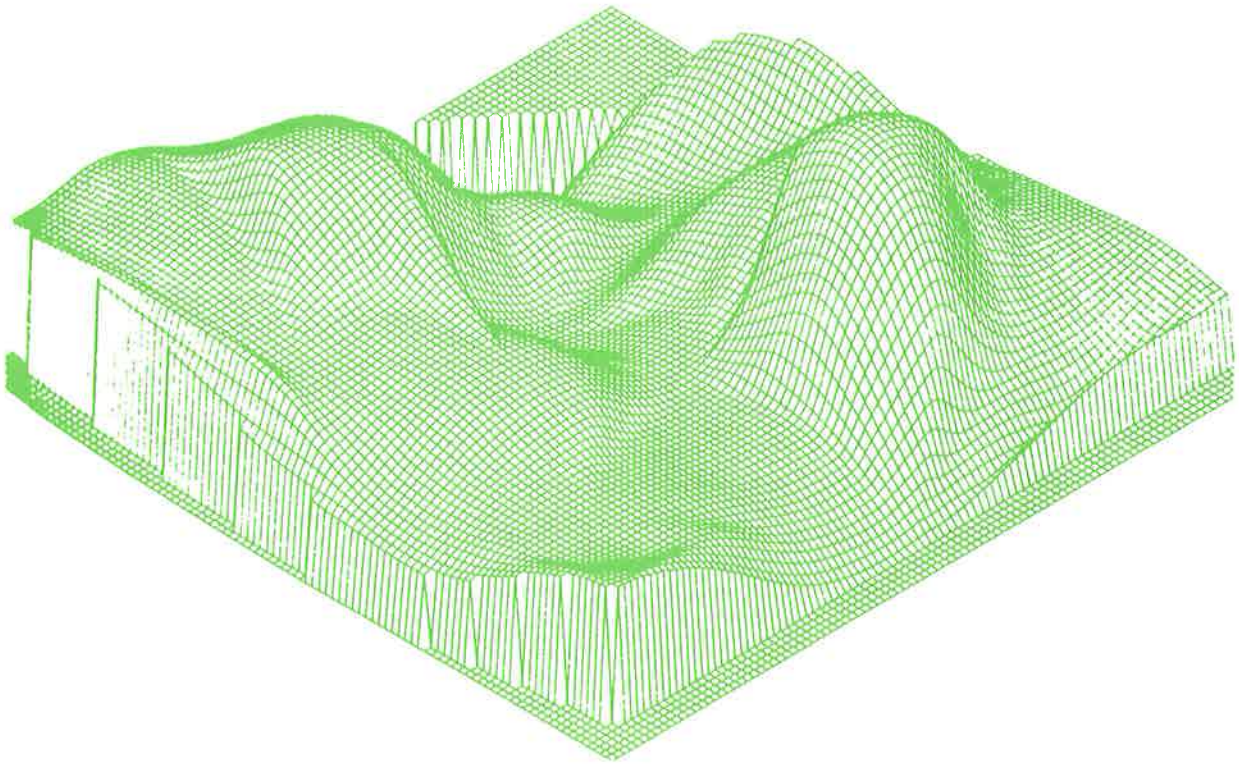
Male domination of mathematics is linked with male domination of the dominant social institutions with which professional mathematical work is tied, most notably the state and the economic system. The connection operates through the education system, through state and corporate funding and through professional and personal contacts (Bowling & Martin 1985).

The high status of mathematics as a discipline may be attributed in part to its image as a masculine area. Mathematical models gain added credibility through the image of mathematics as rational and objective — characteristics associated with masculinity — as opposed to models of reality which are seen as subjective and value-laden.

Specialisation There are various ways in which mathematicians shape and use their expert knowledge to promote their interests *vis-à-vis* other social groups. If mathematical knowledge were too easy to understand by others — both non-mathematicians and other mathematicians — the claims by mathematicians for social resources and privilege would be harder to sustain. Specialisation enables enclaves of expertise to be estab-

lished, preventing scrutiny by outsiders. In applications work, specialisation ensures that only particular groups are served. In all cases, specialisation plus devices such as jargon prevent ready oversight by anybody other than other specialists. Since hiring professionals to understand specialist bodies of knowledge can be afforded on a large scale only by governments and large corporations, specialisation serves their interests more than those of the disabled or the unemployed, for example.

The role of these factors is particularly obvious in mathematical modelling. A mathematical model may be a set of equations, which is thought to correspond to certain aspects of reality. For example, most of theoretical physics, such as elementary theory for projectiles or springs, can be considered to consist of mathematical models. In most parts of physics the models are considered well established, and physicists work by manipulating or adapting the existing models. But in other areas the choice of models is open. Various parts of the reality may be chosen as significant, and various mathematical tools may be brought to bear in the modelling process.



Many people who have been involved in mathematical modelling will realise the great opportunities for building the values of the modeller into the model. I have seen this process at work in a variety of areas, including mathematical ecology, game theory, stratospheric chemistry and dynamics, voting theory, wind power and econometrics.

A good example is the systems of difference equations used in the early 1970s to determine the 'limits to growth'. The choice of equations and parameters more or less ensured that global instability would result (Cole *et al.* 1973). When different assumptions were used by different modellers, different results — for example, that promotion of global social equality would prevent global breakdown — were obtained, nicely compatible with the values of the modellers. Another example is the values built into the global energy projections developed at the International Institute for Applied Systems Analysis (Keepin & Wynne 1984).

Mathematical models are socially significant in two principal ways: as practical applications of mathematics and as legitimations of policies or practices. Most models are closely tied to practical applications, such as in industry. The narrow specialisation involved in the modelling ensures that few other than those developing or funding the application would be interested in or capable of using the model. This sort of applied mathematics is closely linked to the social interests making the specific application. Whether the application is telecommunications satellites, anti-personnel weapons or solar house design, one may judge the mathematics by the same criteria used to judge that application. It is not adequate to say that the killer is guilty while the murder weapon is innocent, for in these sorts of applications the mathematical 'weapon' is especially tailored for its job. Certainly applied mathematicians cannot escape responsibility for their work by referring to 'neutral tools', whether this refers to their mathematical constructions or to themselves.

Models serving as legitimations are involved in a more complicated dynamic. In many cases such as limits-to-growth studies the models do no more than 'mathematicise' a conclusion which would be obvious without the model. But the models are seen as important precisely because they are mathematical, thus drawing on the image of mathematics as objective. A mathematics-based claim also has the advantage of being the work of professionals. Anyone can make a claim, but if a *scientist* does so, relying on the allegedly objective tools of mathematics, that is much more influential. Although exercises in mathematical modelling are often shot through with biases, for public consumption this often overlooked; the modellers draw on an aura of objectivity which is sutained by the more esoteric researches of pure mathematicians.

What then of pure mathematics? There are two major ways in which a link to social interests can be made. First is *potential* applications. These are not always easy to assess, but a good guess often can be obtained by looking at actual applications in the same or related specialities. If any new application turns up, it is likely to be in the same areas and to be used by the same groups.

It is a debatable point whether mathematics should ever be evaluated separately from applications. Arguably, the study of nature is the primary motivation for the

development of and importance of mathematics, and the 'correctness' of pure mathematics should be judged by its ultimate applicability to the physical world (Kline 1959, 1980). The primary reason for the ascension of pure mathematics, namely mathematics which is isolated from application, is the social system of modern science.

This system — including funding, professionalisation, male domination and specialisation — in which claims to sole authority over areas of knowledge are used to claim social resources, is the second way that pure mathematics is connected with social interests. Even if some bit of pure mathematical research turns out to have no application, it is still usually the case that social resources have been expended to support professional workers who are mostly male and who produce intellectual results of interest only to a handful of others like themselves. Furthermore, the work of pure mathematicians, and indeed their very existence, helps legitimate the claims of mathematics to objectivity.

Conclusions

The question, 'What is the link between mathematics and social interests?', is usually answered in advance by assumptions about what *mathematics* really is. If mathematics is taken to be that body of mathematical knowledge which sits above or outside of human interests, then by definition social interests can only be involved in the practice of mathematics, not in *mathematics* itself. This Platonic-like conception sees mathematics as value-free, but is itself a value-laden conception: it serves to deflect attention from the many links between mathematics and society.

Most people would agree that nuclear weapons have not been constructed to serve all people equally; particular social interests are involved in designing, building, testing and deploying nuclear weapons. But what of the uranium, plutonium, iron and other atoms contained in nuclear weapons? Are these atoms 'value-laden'? A reasonable stance in my view is that the atoms in themselves are not linked to any particular groups — except the plutonium atoms which were manufactured by humans — but that the connection enters through the humanly constructed configuration of atoms. The idea of a value-free atom in isolation is all very well, but that is not what we encounter in human constructions.

Elements of mathematical knowledge can be likened to atoms, except that all mathematical concepts have been created by humans. In isolation, the mathematical concepts of an integral or a ring seem not to be associated with the interests of particular groups in society. But mathematical concepts do not exist in isolation. They are organised together for particular purposes, very narrowly for detailed applications, more generally for teaching. The more specialised and advanced ideas are mostly restricted to a small segment of the population, which claims social resources and status due to its expertise.

The belief that mathematics is a body of truth independent of society is deeply embedded in education and research. This situation, by hiding the social role of mathematics behind a screen of objectivity, serves those groups which preferentially benefit from the present social system of mathematics. Exposing the links between mathematics and social interests should not be seen as a threat to 'mathematics' but rather as a threat to the groups that reap without scrutiny the greatest material and ideological benefits from an allegedly value-free mathematics.

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